A Mathematical Model of Drum-type Boilers with Thermal Stress Calculation in the Circulating Fluidized Bed Combustor

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1. Introduction

A CFBC boiler power plant includes four main parts which are the combustor, the boiler, the turbine, and the generator system.

In this study, a nonlinear dynamic model for drum-type boiler is developed.

To simulate water circle through the drum-type boiler system

To simulate the heat transfer process from steam to drum shell

To compute the stresses generating in metal wall due to the thermal gradient inside the metal and pressure.

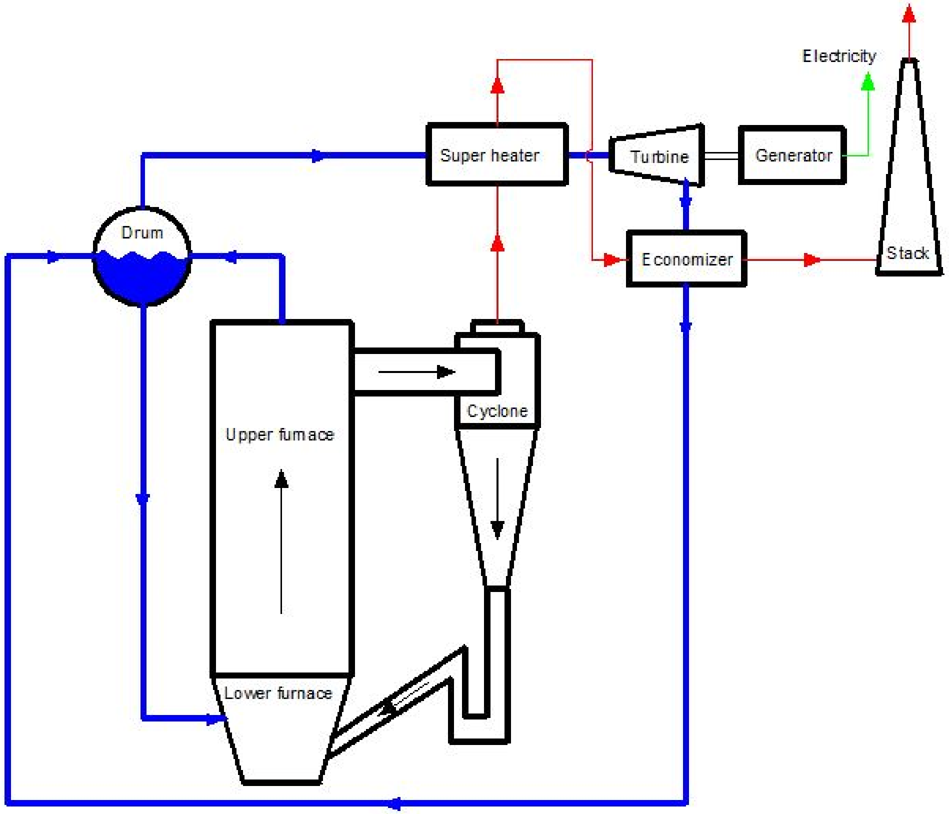


Figure 1. Schematic diagram of the circulating fluidize bed combustion boiler power plant

2. Model

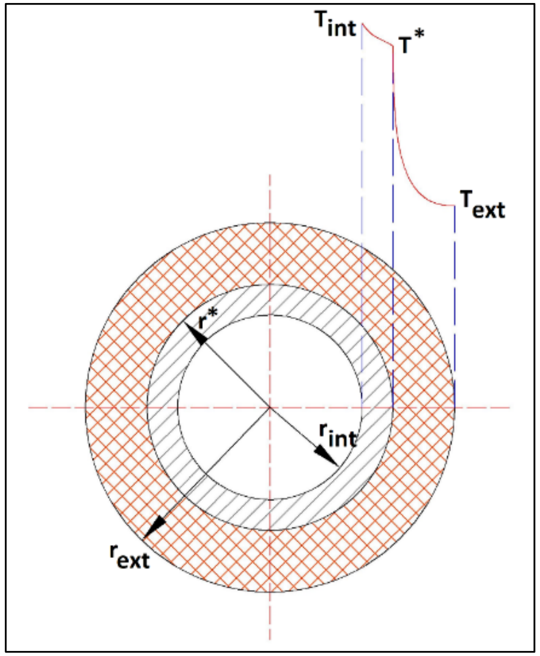


Figure 3. Schematic picture of the drum shell

There is a circulation fluid flow loop through the components in drum-boiler system

The heat transfers from hot steam to wall and insulation.

Based on principle conservation laws, the fluid dynamic model and thermal model are combined to establish the drum-type boiler model.

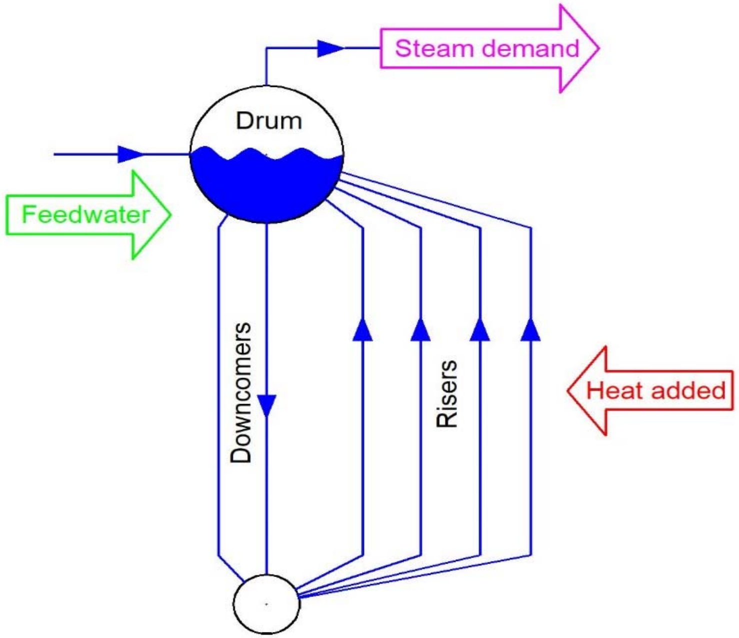


Figure 2. Schematic picture of the drum-boiler system

2.1 Mathematical fluid model

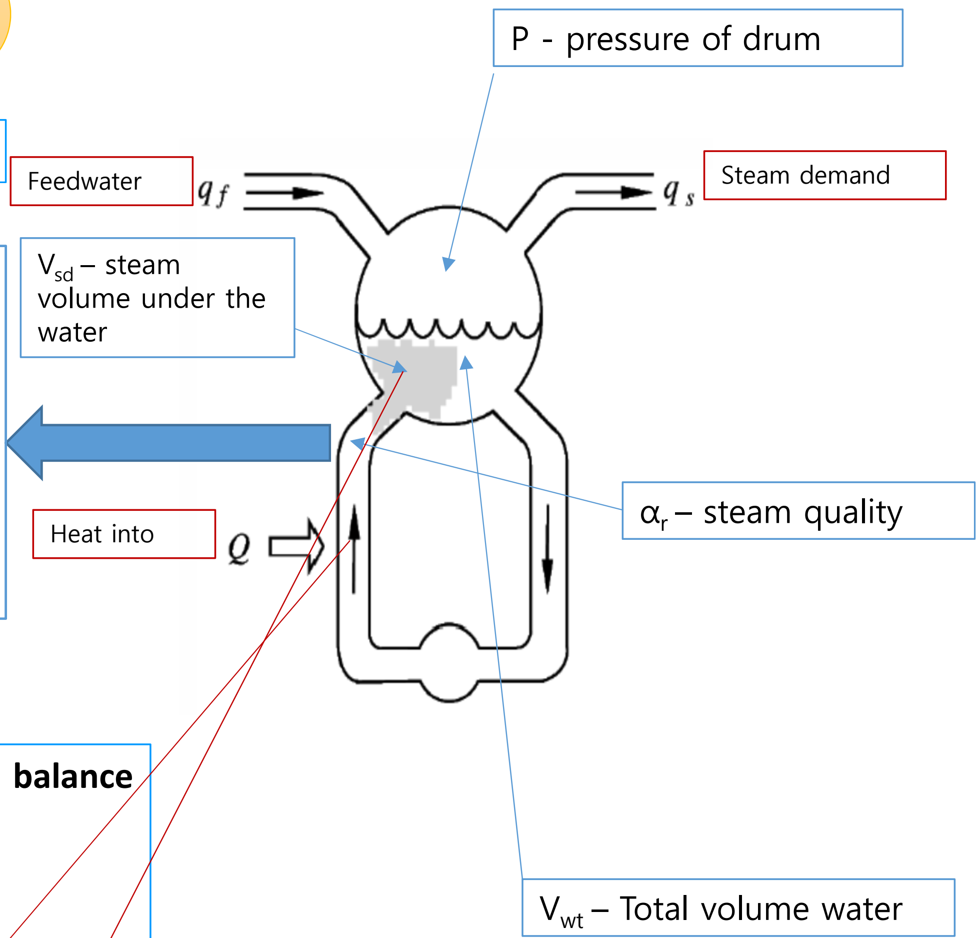
- **The mathematical model of natural circulation drum-boiler.**

$$e_{11}\frac{dV_{wt}}{dt} + e_{12}\frac{dp}{dt} = q_f - q_s,$$

$$e_{21}\frac{dV_{wt}}{dt} + e_{22}\frac{dp}{dt} = Q + q_f h_f - q_s h_s,$$

$$e_{32}\frac{dp}{dt} + e_{33}\frac{d\alpha_r}{dt} = Q + q_{dc} h_c \alpha_r,$$

$$e_{42}\frac{dp}{dt} + e_{43}\frac{d\alpha_r}{dt} + e_{44}\frac{dV_{sd}}{dt} = \frac{\rho_s}{T_d}(V_{sd}^0 - V_{sd}) + \frac{1}{h_c}(h_f - h_w)q_f,$$



- **The dynamic model is captured by mass and energy balance equations of three sub-system which are:**

1. **The global mass and energy balances of whole system**
2. **The mass and energy balances of riser section**
3. **The mass balance of the steam under the water in the drum**

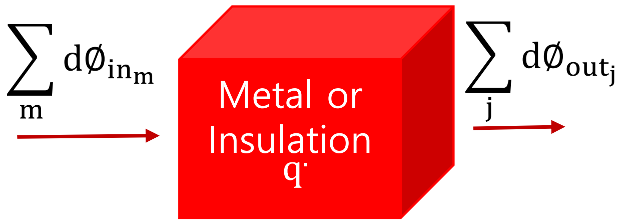
Figure 5. Fluid dynamics model

2.2 Mathematical thermal model

1. Assumptions

- Heat convection from the steam to the inside wall of the drum is neglected.
- The internal energy sources of the material are null.
- The ratio between the drum length and its diameter is assumed sufficiently high therefore the only radial variation of the heat flux and temperature is considered.

2. Energy Balance



$$q^{\circ} = \sum_m d\emptyset_{in_m} - \sum_j d\emptyset_{out_j}$$

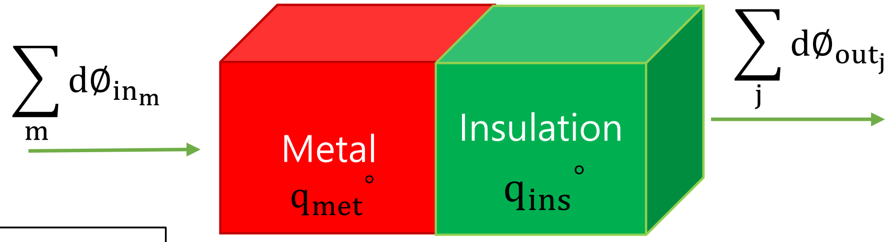
Figure 6. Energy balance model for single uniform material layer

Where:

$\sum_m d\emptyset_{in_m}$: Sum of the inlet thermal flows.

$\sum_j d\emptyset_{out_j}$: Sum of the outlet thermal flows.

$q^{\circ} = c\rho dV\frac{\partial T}{\partial t}$: Rate of heat accumulation within the controlled volume.



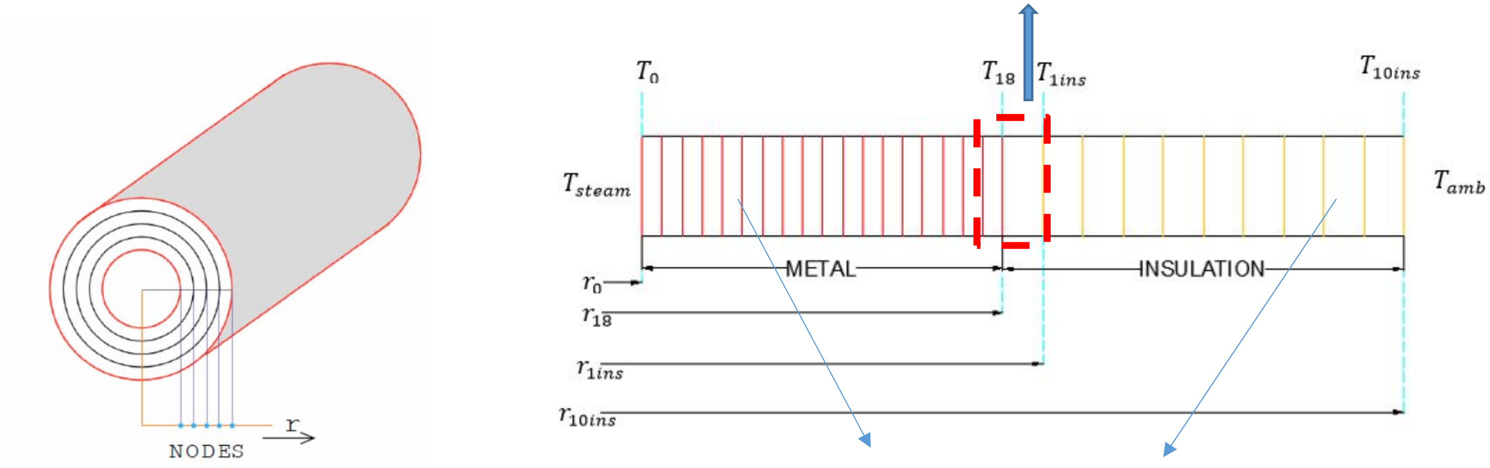
$$q_{met}^{\circ} + q_{ins}^{\circ} = \sum_m d\emptyset_{in_m} - \sum_j d\emptyset_{out_j}$$

Figure 7. Energy balance model for both metal wall layer and insulation layer

2.2.1 Discretized method

The models are solved by finite difference method (FDM) for both steady-state and transient conditions using Matlab codes.

$$\left(\frac{k_{met}}{a_{met}} + \frac{k_{ins}}{a_{ins}}\right)\frac{\partial T}{\partial t} = -k_{met}.\frac{\partial T}{\partial r}.\frac{1}{dr} + k_{ins}.\frac{\partial T}{\partial r}.\frac{1}{dr} + \frac{k_{ins}}{r}.\frac{\partial T}{\partial r}. + k_{ins}\frac{\partial^2 T}{\partial r^2}$$



$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}.\frac{\partial T}{\partial r} = \frac{1}{a}\frac{\partial T}{\partial t}$$

Figure 8. The discretization of the drum' shell

The metal wall is discretized by 18 even layers and the insulation cover is divided into 10 equal cells.

2.2.2 Discretized energy balance equation

Based on the backward and forward for partial derivatives, the energy balance equation for both metal wall and insulation is discretized by equations as follows

$$
\begin{cases}
T_1^{t+\Delta t} = T_1^t + \dfrac{a.\Delta t}{(\Delta r)^2}.\left[T_2^t.\left(1+\dfrac{\Delta r}{r_1}\right) - T_1^t.\left(2+\dfrac{\Delta r}{r_i}\right) + T_0^t\right] \\
T_2^{t+\Delta t} = T_2^t + \dfrac{a.\Delta t}{(\Delta r)^2}.\left[T_3^t.\left(1+\dfrac{\Delta r}{r_2}\right) - T_2^t.\left(2+\dfrac{\Delta r}{r_2}\right) + T_1^t\right] \\
. \\
. \\
. \\
T_{18}^{t+\Delta t} = T_{18}^t + \Delta t.\dfrac{\left[-k_{met}.\dfrac{T_{18}^t - T_{17}^t}{\Delta r_{met}}.\dfrac{1}{\Delta r_{met}} + k_{ins}.\dfrac{T_{1\_ins}^t - T_{18}^t}{\Delta r_{ins}}.\dfrac{1}{\Delta r_{ins}} + \dfrac{k_{ins}}{r_{18}}.\dfrac{T_{1\_ins}^t - T_{18}^t}{\Delta r_{ins}} + k_{ins}.2.\left[\dfrac{T_{1\_ins}^t - T_{18}^t}{\Delta r_{ins}.(\Delta r_{ins}+\Delta r_{met})} - \dfrac{T_{18}^t - T_{17}^t}{\Delta r_{met}.(\Delta r_{ins}+\Delta r_{met})}\right]\right]}{\left(\dfrac{k_{met}}{a_{met}}.+\dfrac{k_{ins}}{a_{ins}}\right)} \\
T_{1\_ins}^{t+\Delta t} = T_{1\_ins}^t + \dfrac{a.\Delta t}{(\Delta r)^2}.\left[T_{2\_ins}^t.\left(1+\dfrac{\Delta r}{r_{1\_ins}}\right) - T_{1\_ins}^t.\left(2+\dfrac{\Delta r}{r_{1\_ins}}\right) + T_{18}^t\right] \\
T_{2\_ins}^{t+\Delta t} = T_{2\_ins}^t + \dfrac{a.\Delta t}{(\Delta r)^2}.\left[T_{3\_ins}^t.\left(1+\dfrac{\Delta r}{r_{2\_ins}}\right) - T_{2\_ins}^t.\left(2+\dfrac{\Delta r}{r_{2\_ins}}\right) + T_{1\_ins}^t\right] \\
. \\
. \\
. \\
T_{9\_ins}^{t+\Delta t} = T_{9\_ins}^t + \dfrac{a.\Delta t}{(\Delta r)^2}.\left[T_{10\_ins}^t.\left(1+\dfrac{\Delta r}{r_{9\_ins}}\right) - T_{9\_ins}^t.\left(2+\dfrac{\Delta r}{r_{9\_ins}}\right) + T_{8\_ins}^t\right]
\end{cases}
\quad (7)
$$

2.3. Stress distribution

Thermal stress distribution

Referenced by Kim, T. S., Lee, D. K., Ro, S. T., 2000, Analysis of thermal stress evolution in the steam drum during start-up of a heat recovery steam generator, Applied Thermal Engineering, vol.20, pp. 977 – 992, 2000

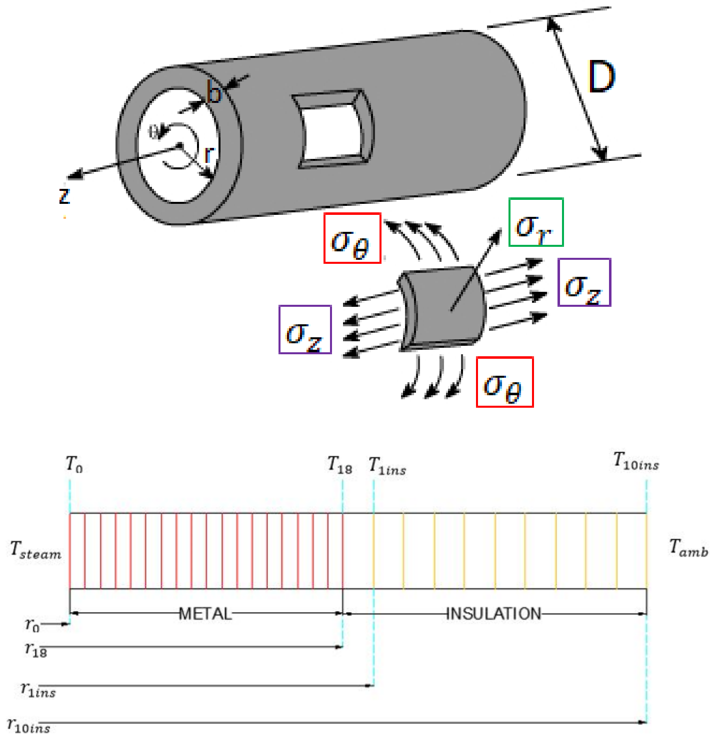


Figure 9. The thermal stress model of metal drum wall

Tangential stress:

$$\sigma_{\theta-T} = \frac{E.\alpha}{(1-v).r^2}.\left[\frac{r^2 + r_{int}^2}{r^{*2} - r_{int}^2}.\int_{r_{int}}^{r^*} T(r).r.dr + \int_{r_{int}}^{r} T(r).r.dr - T(r).r^2\right] \quad (8)$$

Radial stress:

$$\sigma_{r-T} = \frac{E.\alpha}{(1-v).r^2}.\left[\frac{r^2 - r_{int}^2}{r^{*2} - r_{int}^2}.\int_{r_{int}}^{r^*} T(r).r.dr - \int_{r_{int}}^{r} T(r).r.dr\right] \quad (9)$$

Longitudinal stress:

$$\sigma_{z-T} = \frac{E.\alpha}{(1-v)}.\left[\frac{2}{r^{*2} - r_{int}^2}.\int_{r_{int}}^{r^*} T(r).r.dr - T(r)\right] \quad (10)$$

Effective thermal stress can be calculated by von-Misses theory as equation follows

$$\sigma_{eff-T} = \left[\sigma_{\theta-T}^2 + \sigma_{r-T}^2 + \sigma_{z-T}^2 - (\sigma_{\theta-T}\sigma_{r-T} + \sigma_{\theta-T}\sigma_{z-T} + \sigma_{r-T}\sigma_{z-T})\right]^{1/2}$$

Where:

Modulus of elasticity: E = 207 Gpa

Thermal expansion: α = $1.4.10^{-6}$ K^{-1}

Poisson's ratio: v = 0.292

Mechanical stress distribution

The combine mechanical stresses and thermal stresses

Tangential stress: $\sigma_\theta = \sigma_{\theta-T} + \sigma_{\theta-P}$

Radial stress: $\sigma_r = \sigma_{r-T} + \sigma_{r-P}$

Longitudinal stress: $\sigma_l = \sigma_{l-T} + \sigma_{l-P}$

The mechanical stresses are caused by pressure

Tangential stress: $$\sigma_{\theta-P} = \frac{r_i^2 * P}{(r_o^2 - r_i^2)}\left[1 + \frac{r_o^2}{r^2}\right]$$

Radial stress: $$\sigma_{r-P} = \frac{r_i^2 * P}{(r_o^2 - r_i^2)}\left[1 - \frac{r_o^2}{r^2}\right]$$

Longitudinal stress: $\sigma_{l-P} = 0$

Effective mechanical stress can be calculated by von-Misses theory as equation follows

$$\sigma_{eff-P} = \left[\sigma_{\theta-P}^2 + \sigma_{r-P}^2 + \sigma_{z-P}^2 - (\sigma_{\theta-P}\sigma_{r-P} + \sigma_{\theta-P}\sigma_{z-P} + \sigma_{r-P}\sigma_{z-P})\right]^{1/2}$$

Effective stress can be calculated by von-Misses theory as equation follows

$$\sigma_{eff} = \left[\sigma_\theta^2 + \sigma_r^2 + \sigma_z^2 - (\sigma_\theta\sigma_r + \sigma_\theta\sigma_z + \sigma_r\sigma_z)\right]^{1/2}$$

2.4. Solution procedures

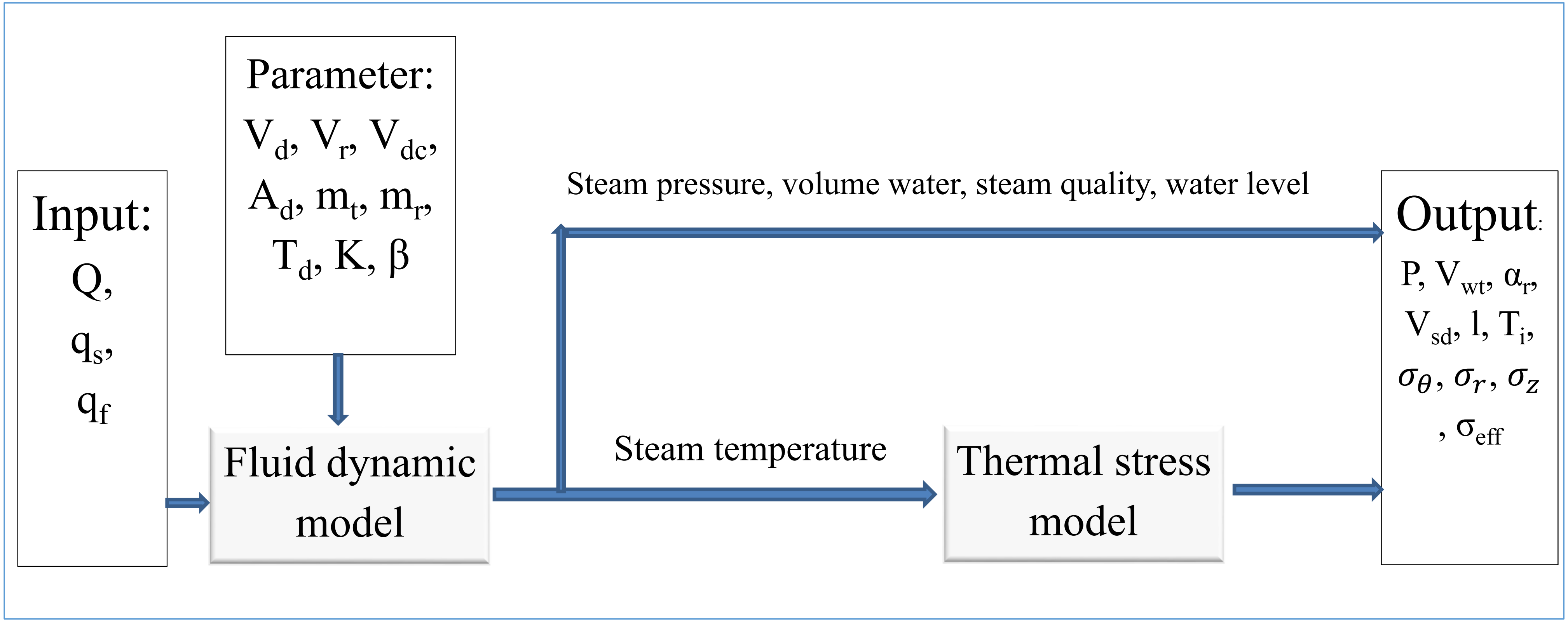
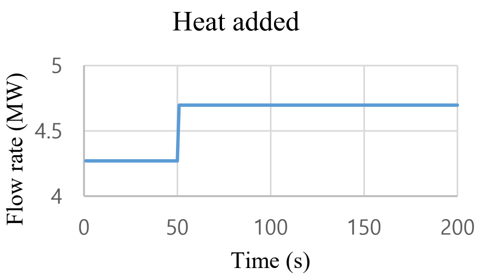
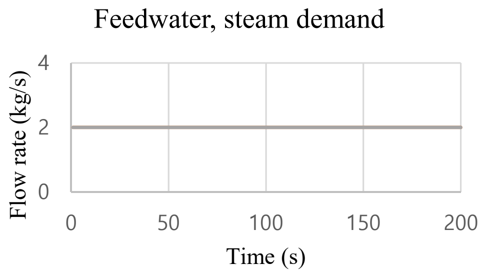


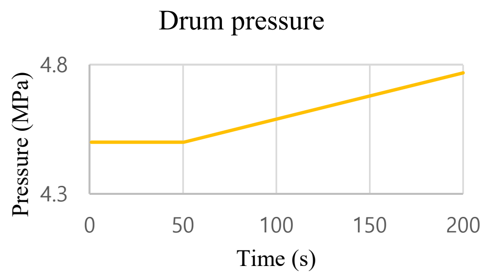
Figure 10. The solution procedures of the drum-type boiler model

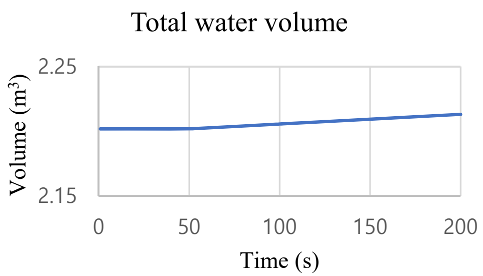
3. Simulation results
3.1. Step change

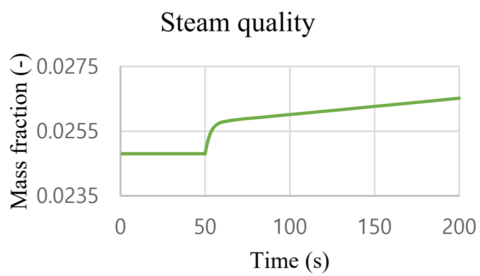
3.1.1 A step change 10% value in heat addition flow rate.

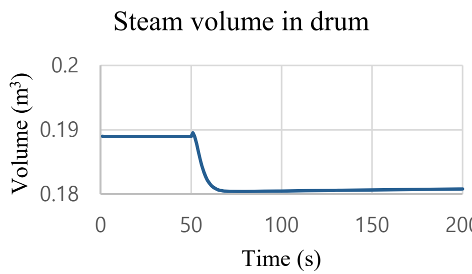


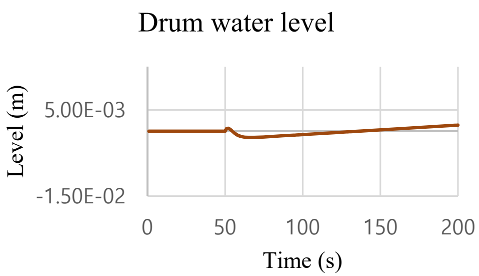


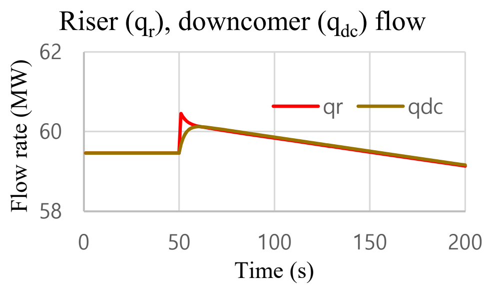


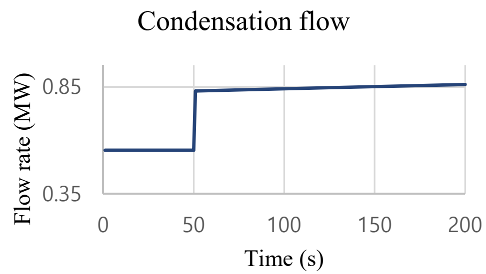




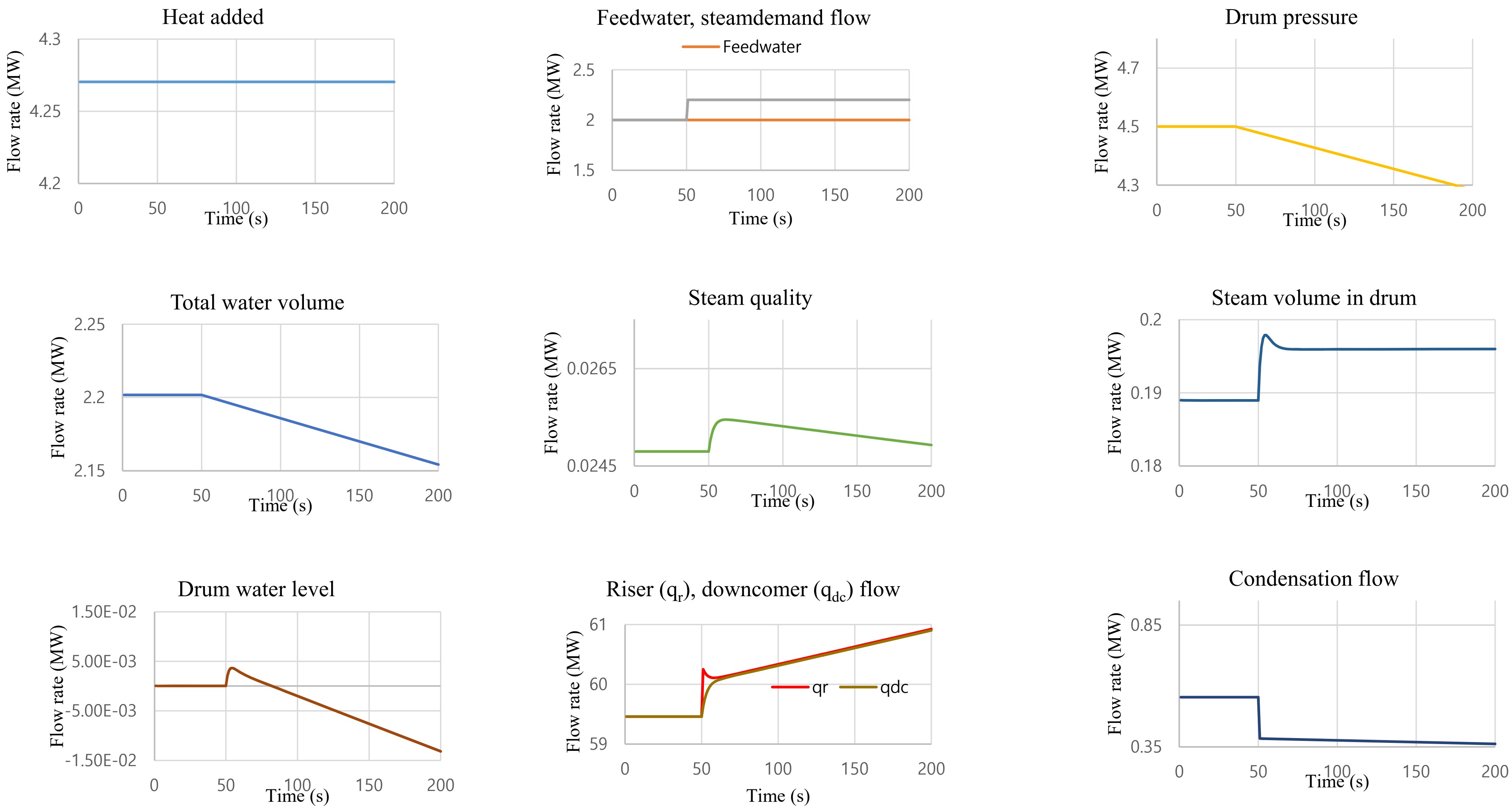




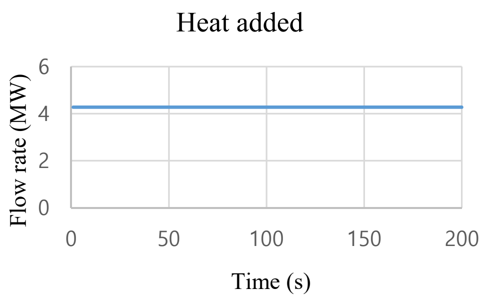


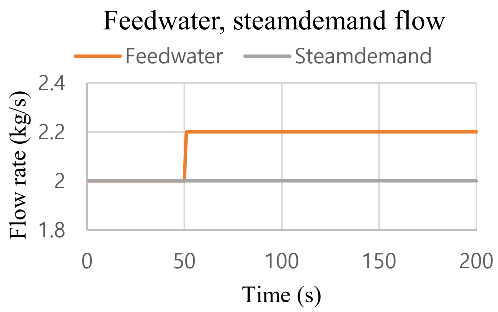


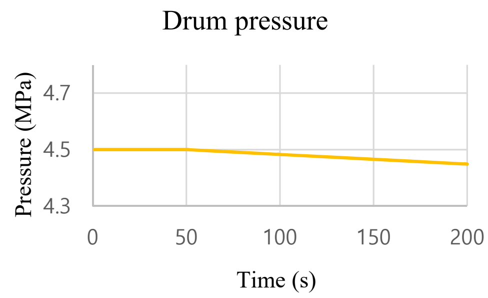
3.1.2 A step change 10% value in steam demand flow rate.

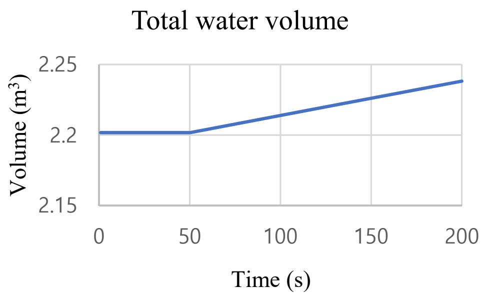


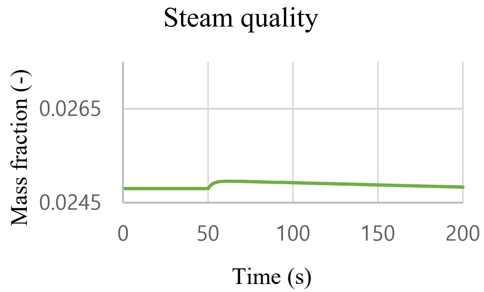
3.1.3 A step change 10% value in feedwater flow rate.

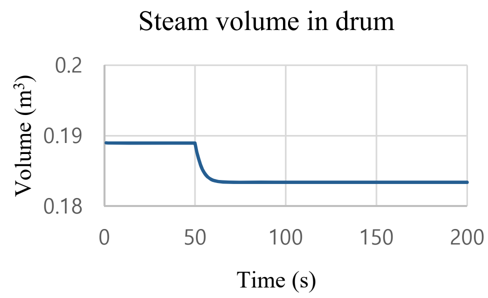


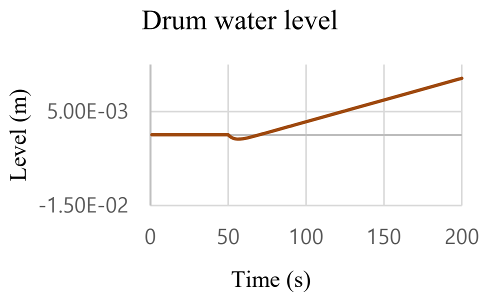


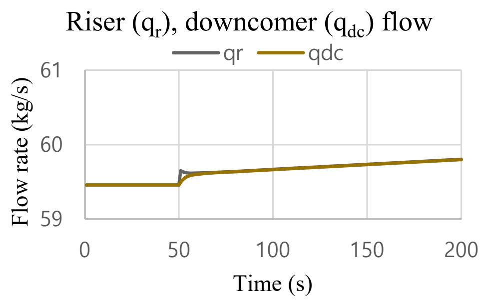


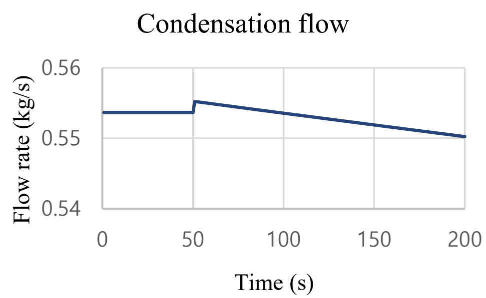




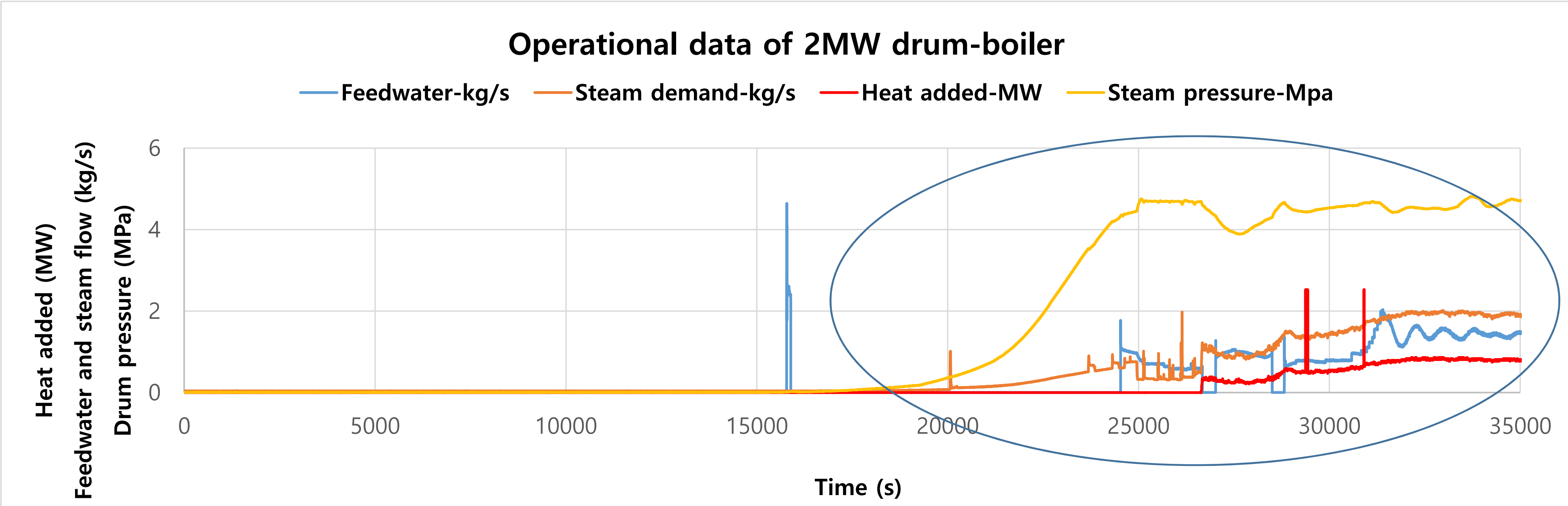








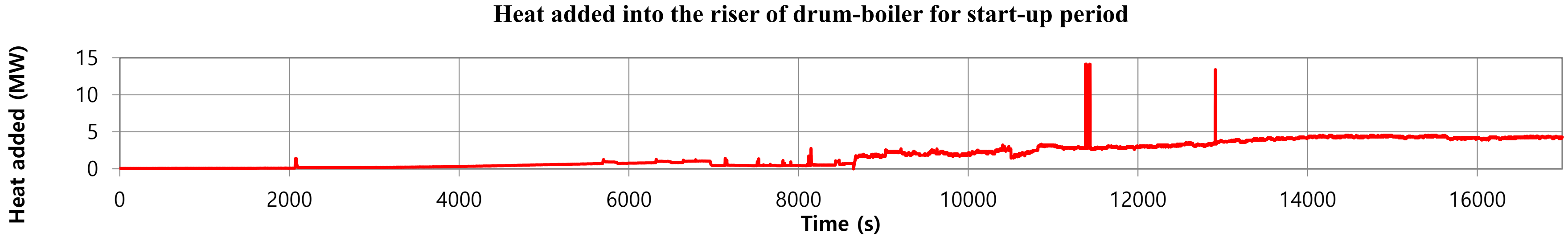
3.2 Start-up period

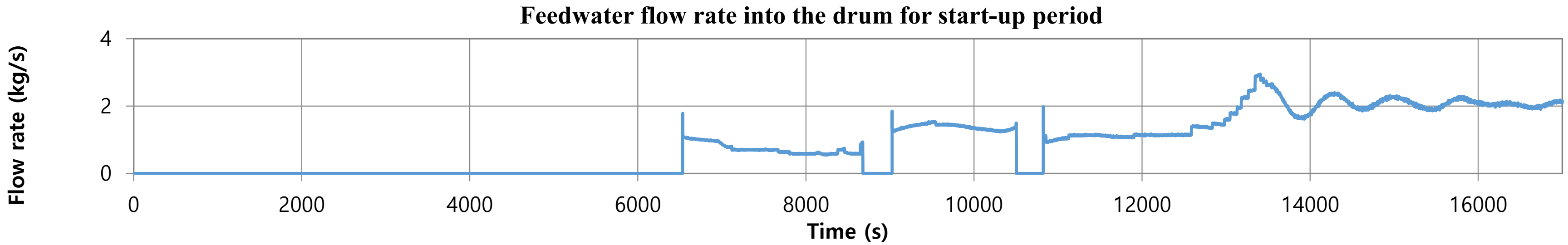


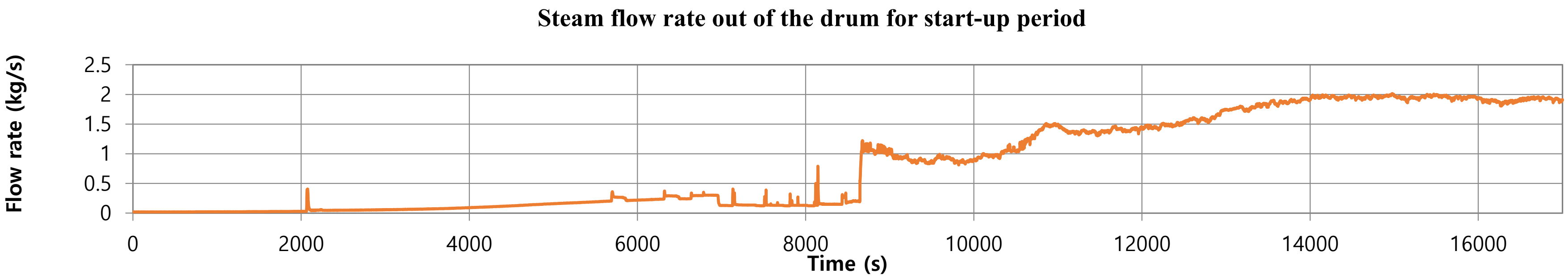
The start-up period is from 18,000s to 35,000s

3.2.1. Fluid dynamics

3.1.1. The input data of 2 MW drum-boiler for start-up period

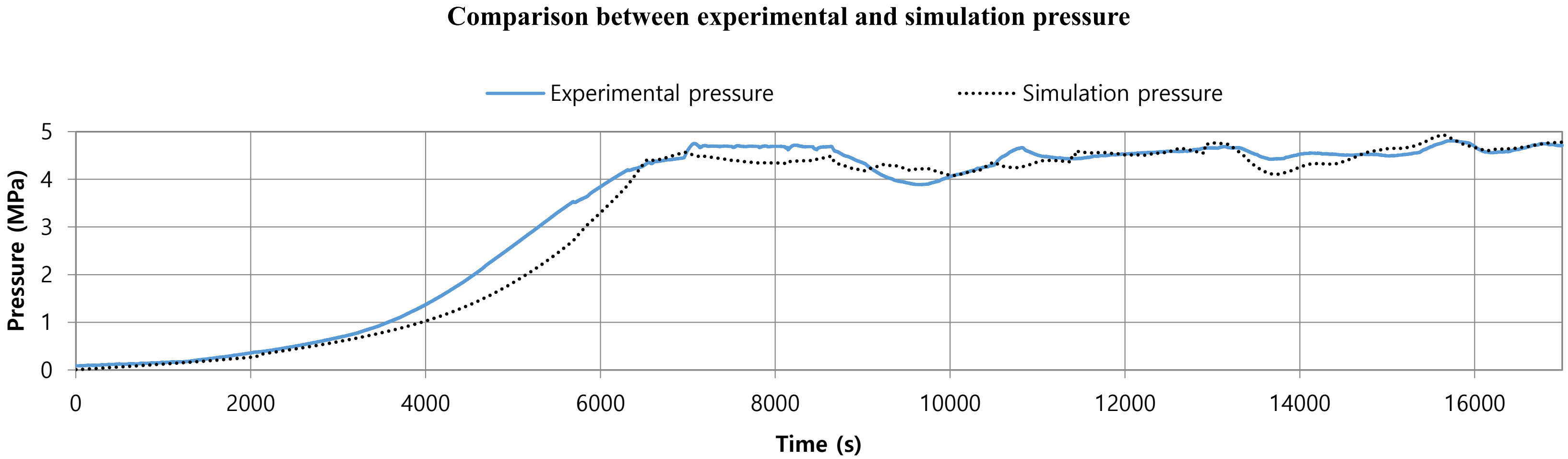






3.1.2. The simulation results of 2 MW drum-boiler during start-up period

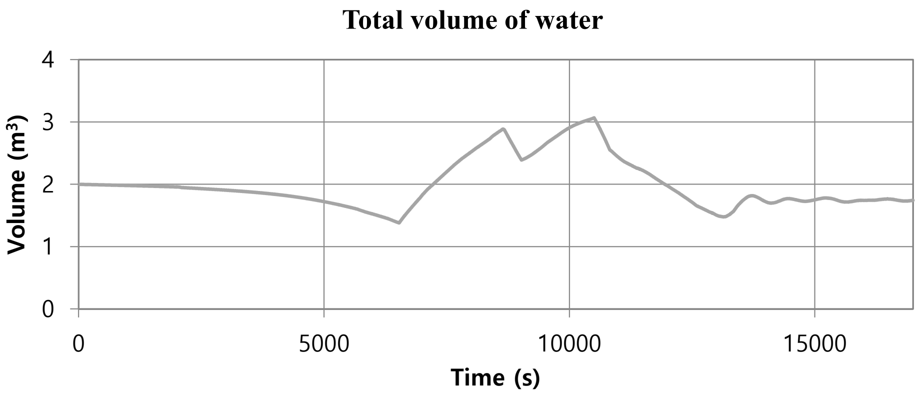
Drum pressure

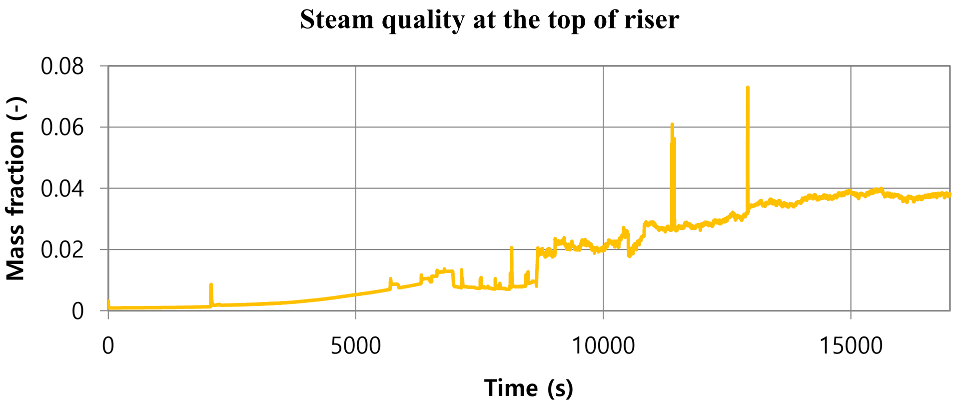


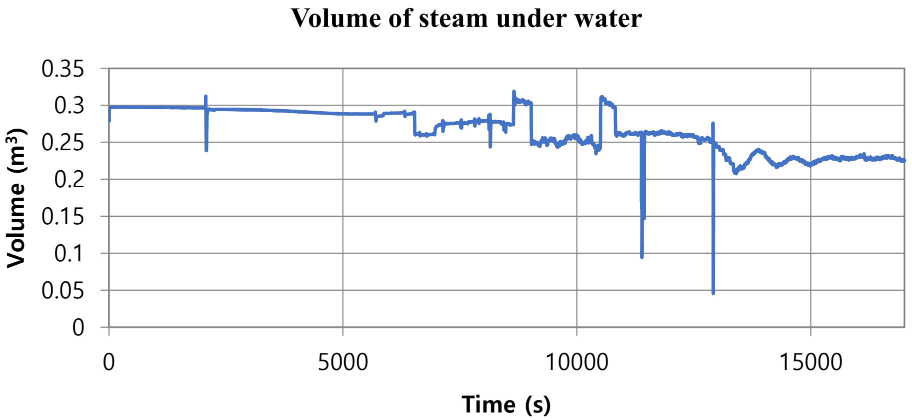
There are a few differences from time 3,500s to 6,300s because this interval time the effect of the efficiency of drum-boiler is the most pronounced.

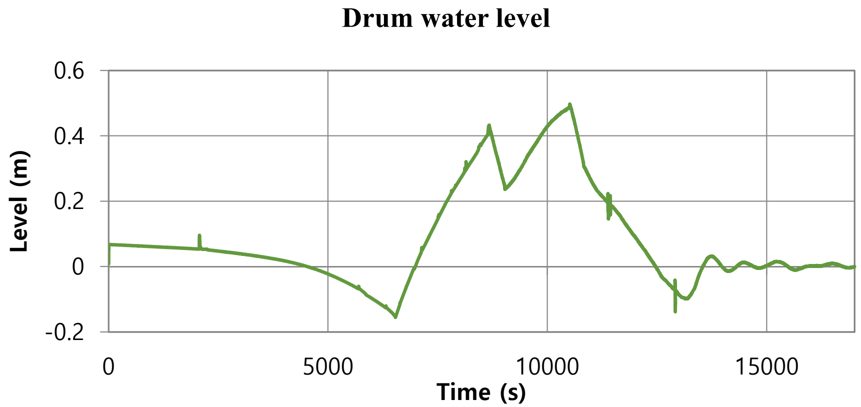
In the interval time from 14,000s to 17,000s, there is a pretty good agreement between the model and experimental data because the system reaches nearly stable operation.

Other simulation results for start-up period without validation

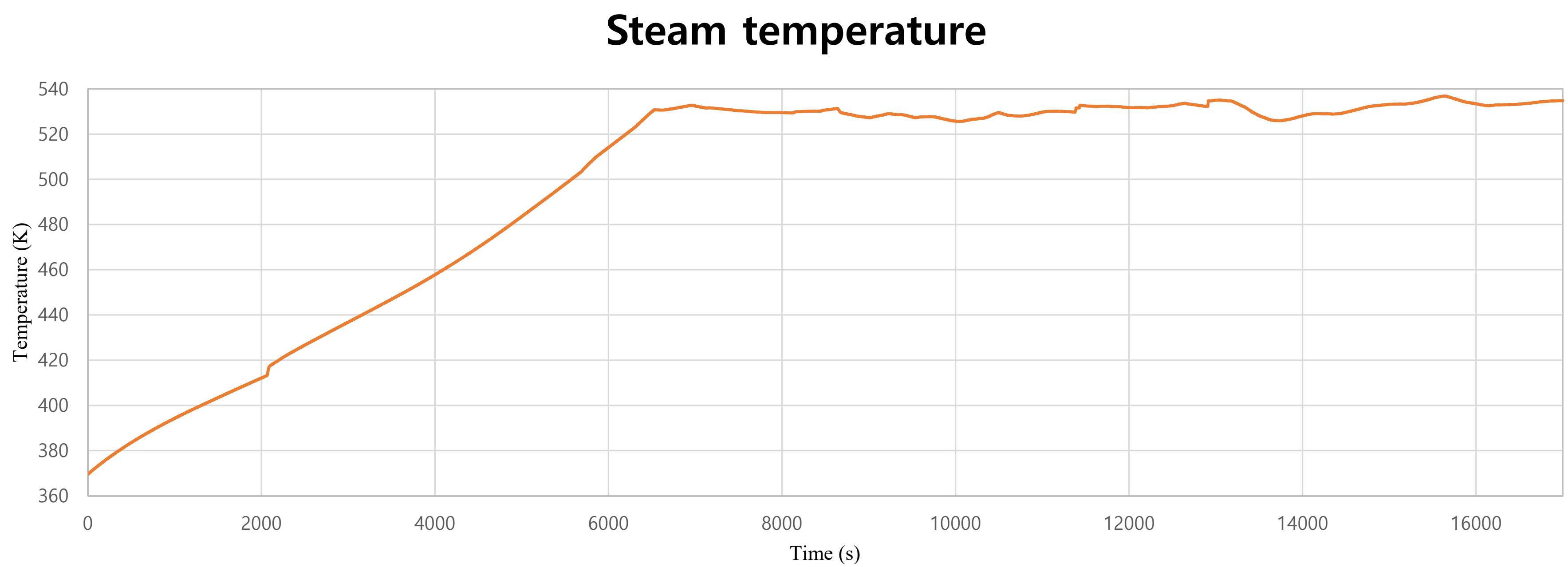




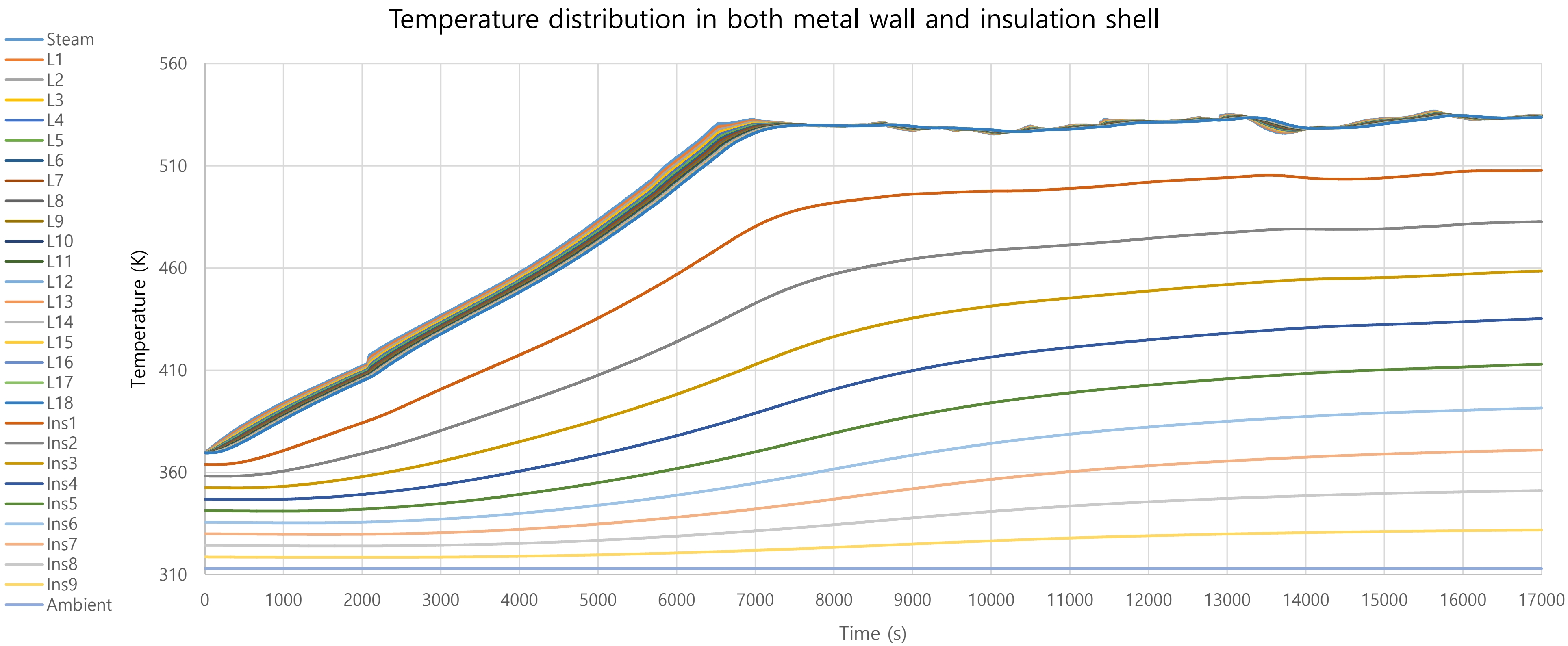




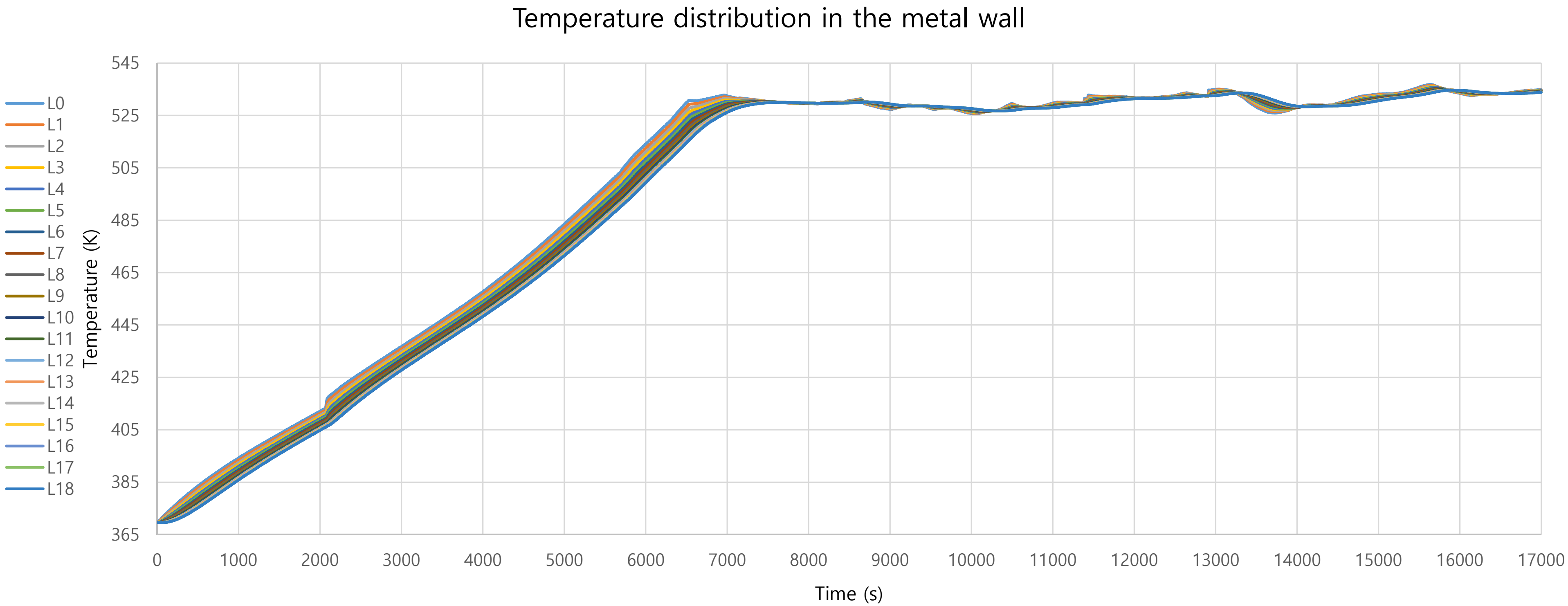
3.2.2. Temperature distribution



Based on the simulation results in above section, the fluid dynamic sub-model, the temperature history during start-up period is computed in order to examine the variation of the temperatures and thermal stresses generated inside metal wall of steam drum.

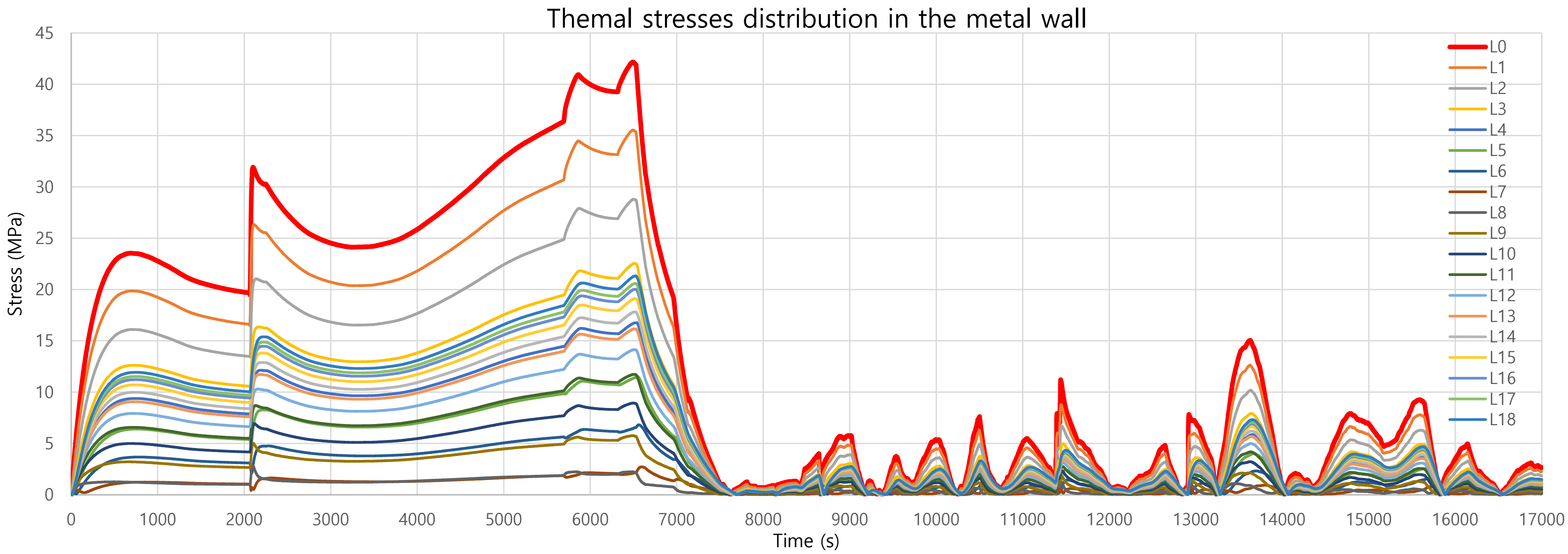


The outer insulation temperature is a boundary condition therefore with steam temperature is given in above section. The temperature distribution in 18 layer of the metal wall and the temperature distribution in 10 layer of the insulation shell are simulated and shown together in Figure.

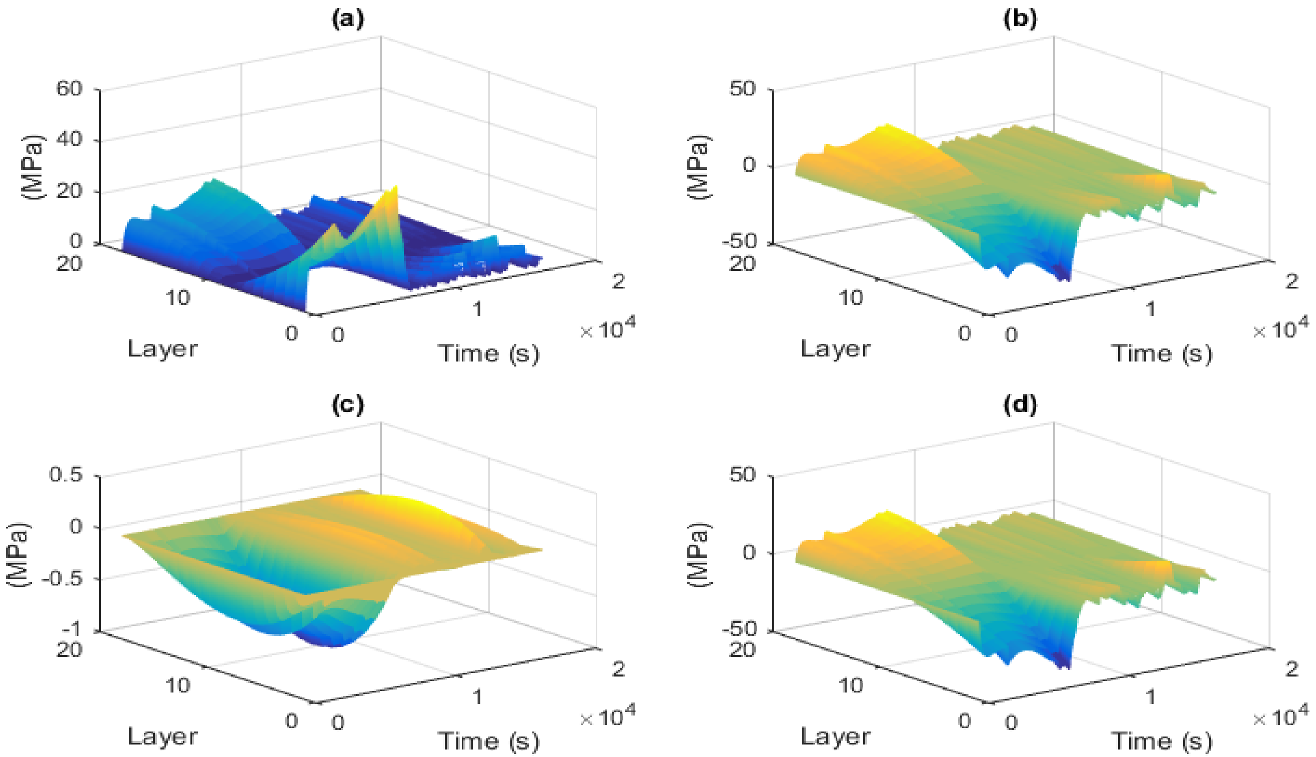


The temperature distribution in metal wall is shown in figure which indicate that during the steam temperature increases the temperature in layer 1 is highest and the temperature in layer 18 is lowest and vice versa for the steam temperature decreases process. These phenomena occurred because of the heat transfer process between steam and metal wall.

3.2.3 Stresses distribution



Because of the thermal gradient inside the metal wall, very high thermal stresses are originated. As shown by figure, the simulation results indicate that the von Mises equivalent stresses reaches the maximum values in the first metal layer which contact with the steam



Direction thermal stresses distribution in the metal wall: (a) Equivalent stress; (b) Tangential stress; (c) Radial stress; (d) Longitudinal stress

4. Conclusion

- A mathematical drum-type boiler model was developed for simulating the drum-boiler dynamics.

- A thermal-stress sub-model of drum shell was developed for simulating the heat transfer through the metal and the insulation of the drum steam.

- A computation model of the thermal and mechanical stresses was proposed for simulating the generated stresses distribution inside the metal drum wall.

- The dynamics of 2MW CFBC boiler was simulated for start-up regimes by using such a model.

- The simulation result of steam pressure was validated against the experimental measurement.

- Other numerical results including total volume of water, steam quality at the top of the riser, the volume of steam under the water, and the drum water level are also well captured by the proposed modeling.

THANK YOU VERY MUCH