Purdue professor solves 140-year fluid mechanics enigma

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Sangtae Kim
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WEST LAFAYETTE, Ind. – A Purdue University researcher has solved a 140-year-old enigma in fluid mechanics: Why does a simple formula describe the seemingly complex physics for the behavior of elliptical particles moving through fluid?

The findings have potential implications for research and industry because ellipsoid nanoparticles are encountered in various applications including those involving pharmaceuticals, foods and cosmetics.

Like a sphere, the oblong ellipsoids undergo "rigid body motion" when submerged in a fluid, meaning they do not deform while moving from side to side and rotating. However, because an ellipsoid is not perfectly spherical, it is counterintuitive that its rigid-body motion in a fluid could be described using the same simple mathematical expression as spheres.

"In general, you would expect a very complicated expression because an ellipsoid is not a perfect sphere," said Sangtae Kim, a distinguished professor in Purdue's School of Chemical Engineering.

Yet, that is not the case, presenting a quandary that Kim has been pondering since his days as an undergraduate in the 1970s.

"It's been gnawing at me since then," he said.

MIT chemical engineer Howard Brenner wrote a paper in 1964 showing the mathematics behind the simple formula, but it took pages of complex calculations to arrive at the simple result.

"Dr. Brenner highlighted this simplicity," Kim said. "But the simplicity of the result could only be shown by going through five to 10 pages of very messy algebra and calculations. In the end, everything cancels and you get this final very simple result. It's almost like a miracle, which has bothered me for a long time."

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He has solved the enigma in a new paper appearing in November in a special issue of the American Chemical Society (ACS) journal Industrial & Engineering Chemistry Research. The special issue honors the 50th anniversary of Purdue's Doraiswami Ramkrishna's doctoral dissertation, which is considered a landmark in the history of chemical engineering. Ramkrishna is Purdue's Harry Creighton Peffer Distinguished Professor of Chemical Engineering. Kim's paper was highlighted as an ACS "editor's choice."

The ellipsoid enigma begins in 1876 and 1892, when scientists described how an ellipsoid moves through surrounding fluid while traveling side-to-side and rotating, respectively, causing pressure and stress on the object's skin referred to as surface traction. Brenner later unified the mathematics for both the side-to-side motion and rotation.

The new research demonstrated how an ellipsoid's interaction with fluid can be described using the same type of simple mathematical pattern that applies to spheres.

"The pattern has been known for 140 years, and the fundamental underlying reason for why this simple pattern has to be true is now apparent because of this new work being published," Kim said.

John Anderson, president emeritus of the Illinois Institute of Technology and a professor of chemical engineering, said, "Dr. Kim's paper definitively finalizes the 140-year development of intriguing relationships among hydrodynamic properties of ellipsoids, relationships that have proven invaluable to theorists trying to model the motion of particles in flowing liquids and electric fields. A fascinating backstory is that Professor Kim maintained his interest in proving the exactness of these useful relationships even during his years in executive management in the pharmaceutical industry and the National Science Foundation. The crucial spark was reignited last fall when he was invited to speak in memory of his late colleague Howard Brenner."

Anderson visited Purdue to give a guest seminar in September.

Professor Henry Power of the University of Nottingham, an expert who provided an important discovery in the field in 1987, said: "Professor Kim's elegant solution also provides a new and efficient way for solving for the motion of these nonspherical particles."

Writer: Emil Venere, 765-494-4709, venere@purdue.edu

Sources: Sangtae Kim, 765-494-5692, kim55@purdue.edu

John Anderson, JohnA@iit.edu

Note to Journalists: A copy of the research paper is available from Emil Venere, 765-494-4709, venere@purdue.edu.

ABSTRACT

Ellipsoidal Microhydrodynamics without Elliptic Integrals and How To Get There Using Linear Operator Theory: A Note on Weighted Inner Products

*Sangtae Kim**

School of Chemical Engineering, Purdue University

In this research note we revisit the topic of microhydrodynamics of an ellipsoid in rigid body motion to arrive at the final resolution of a 140-year-old "mystery" that was featured in the dedication paper on the same topic in the Doraiswami Ramkrishna Festschrift. There, the initial focus was on the role of the theory of self-adjoint operators as the framework for proving that the surface tractions on a sphere had to be a constant multiple of the same rigid body motions of the boundary conditions. The ellipsoid was then considered as a simple example to illustrate the loss of this behavior for nonspherical particles. That goal was accomplished because for an ellipsoid, n·x, the dot product of the surface normal n and the point x on the ellipsoid surface, is the required nonconstant multiplier. The simplicity of this result is striking and has been noticed throughout its history with a number of authors remarking on the lengthy algebraic manipulations required to prove this simple result. In keeping with the theme of the Doraiswami Ramkrishna Festschrift, this note presents a short and simple proof that highlights the importance of the choice of the inner product, that is, the definition of the metric. The introduction of n·x = w(x) as a so-called weight function in the definition of the weighted inner product, as in f, g w = ∫ f(s) g(s) w(s)d s over the appropriate metric space transforms the double layer operator K into a self-adjoint operator. From this it follows that the eigenfunctions of the adjoint with respect to the nonweighted inner product are w times the eigenfunctions of K. Thus, the simplification noted in the companion paper is true for all eigenvalues and eigenfunctions of the double layer operator and not just the eigenvalue of −1 and its associated eigenfunction vRBM. These insights open the door to significant opportunities in the computational analysis of ellipsoidal particles in nanoparticle technology including topics such as perturbation methods for inertial and non-Newtonian effects, as we now have ready access to the spectral decomposition and biorthogonal expansions for the double layer operator.

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Purdue University, 610 Purdue Mall, West Lafayette, IN 47907, (765) 494-4600