8. Chemical consumption per regeneration
   Anion = 2.1 lb. NH₃/cu. ft. loaded in column
   Cation = 5.0 lb. H₂SO₄/cu. ft. loaded in column

9. Water requirements per regeneration, flow rate, and time (air dome operation)
   a. Backwash operation (process water)
      Anion = 20 gal./cu. ft. loaded in column
            at 1.25 gal./min. (cu. ft.)
      Cation = 20 gal./cu. ft. loaded in column
            at 1.25 gal./min. (cu. ft.)
   b. Rinse water (demineralized) to rinse out regenerant chemicals
      Anion = 150 gal./cu. ft. loaded in column
            at 1.0 gal./min. (cu. ft.)
      Cation = 30 gal./cu. ft. loaded in column
            at 1.0 gal./min. (cu. ft.)
   c. New water (demineralized) added to process syrup from rinsing-out syrup at
      start of regeneration procedure and water retained in columns when placing columns on stream:
      Anion = 4.5 gal./cu. ft. loaded in column*
      Cation = 4.5 gal./cu. ft. loaded column*
   d. Water (demineralized) for diluting regenerants:
      Anion = 13 gal./cu. ft. loaded in column;
            pump to column at a rate to give
            40 min. contact time with resin
      Cation = 30 gal./cu. ft. loaded in column;
            pump to column at a rate to give
            40 min. contact time with resin
   e. Time to depress syrup to bed level with air pressure = 10 min.

10. Quality of water available:
    Process water = 80 grains/gal. as Na₂SO₄
    Demineralized water = 2 p.p.m. as Na₂SO₄

11. Free void area for each resin = 30%

*A total of 6.0 gal. of demineralized water/cu. ft. loaded in column is used to rinse out syrup
   ("sweetening off") at start of regeneration procedure. The flow rate for the sweetening-off
   procedure should be 1.0 gal./min. (cu. ft.).

TABLE 3
UNIT COSTS OF UTILITIES AND SUPPLIES

| Steam, saturated 120 and 10 lb./sq. in. gauge | $0.50/1,000 lb. |
| Water Process water | $0.02/1,000 gal. |
| Demineralized water | $0.235/1,000 gal. |
| Sulfuric acid, 96% | $18/ton |
| Anhydrous ammonia | $93/ton |
| Filter aid | $37/ton |

TABLE 4
BAUMÉ - DRY-SUBSTANCE TABLE FOR STARCH SUSPENSIONS AND SYRUP
60°F. / 60°F.

<table>
<thead>
<tr>
<th>Baume gravity</th>
<th>% Dry substance</th>
<th>Pounds/dry substance/gallon</th>
<th>Pounds dry substance/gallon</th>
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<td>1.2612</td>
<td>55.39</td>
<td>10.504</td>
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</tbody>
</table>

SOLUTION

Karl William Rausch, Yale University

After examination of the problem the number of variables involved was narrowed down to three interdependent variables: time, temperature, and the normality of the acid in the converter. Temperature and normality could be varied as desired but both influenced the time. Cost then became dependent upon two independent variables—temperature and normality of the acid. At several different temperatures the total cost for the process was calculated as the normality was varied.

The total costs calculated were graphed vs. the temperature of the converter with normality used as a parameter to obtain several different lines. Each of these lines produced a minimum cost at some temperature. A graph was then made of these minimum values plotted against temperature and normality. Again a minimum point was obtained in both graphs. This minimum point had the same value in both graphs, and it determined the conditions at which the process was least expensive.
Fig. 1. Marked-up flow diagram for manufacturer of dextrose.

Fig. 2.

Fig. 3.
The results of the calculations indicated that the optimum conditions were:
- Converter temperature: 327°F
- Acid normality: 0.010
- Minimum cost/100 lb. of dry substance produced: $0.2012

METHOD

An initial examination shows that there were several variables involved in this process: temperature of reaction, normality of the acid catalyst, time of reaction, and concentration of the starch. As the concentration of dry substance in the exit hydrolysis was specified, it was possible to determine the concentration of the starch in the entering slurry. With this initial concentration, the equilibrium dextrose concentration could be determined from the graph (Figure 1) in the problem statement. This equilibrium concentration and the desired final concentration were sufficient to solve the rate equation for an expression involving time and the rate constant. The rate constant was dependent upon the temperature of the reaction and the normality of the acid. In this way two independent variables were obtained which together determined a third dependent variable.

An analysis of the other steps in the process showed that variations in the cost of these steps was dependent upon either the temperature of the reaction in the converter or the normality of the acid. From this it was deduced that the total cost of the process depended upon these two variables.

The cost of the converter depended upon the length of the converter. The length depended upon the time of reaction, as there was a minimum allowable velocity in the converter. Time in turn depended upon temperature and normality. The cost of the steam depended upon the temperature. Also the amount of water necessary to obtain the desired initial starch concentration from the available slurry was determined by the temperature, because the more steam that was added to the slurry, the less water was necessary. Acid costs naturally depended upon the normality.

The cost of the cooler depended upon the temperature. The higher the temperature, the greater the amount of water vaporized in the cooler and the greater the cost of the cooler.

The filter was dependent only upon the temperature. Higher temperature meant that more water was vaporized in the cooler, hence the more concentrated the hydrolyzate and the greater the necessary filtering surface.

The cost of the cation-exchanging tower was dependent only upon temperature. At high temperatures the cost was determined by the concentration of H.M.F. in the solution. At lower temperatures the sodium sulfate from the water added and from the initial slurry determined the cost of the tower.

The cost of the anion exchanging tower was dependent mostly upon the normality of the acid, although temperature was involved because of the variations in the concentration of sodium sulfate due to temperature.

The cost of each of these individual steps was calculated at several values of the controlling variable. These costs were graphed against this controlling variable. Total costs were then calculated for several normalities at varying temperatures. These total costs were also graphed against temperature. The several normalities formed a series of curves each of which possessed a minimum. The values of these minimum points were then plotted against normality and against temperature. Each of these curves also possessed a minimum value. This indicated that there existed one normality at which the minimum in its temperature-varying curve was lower than that for all other normalities. This lowest minimum was taken to be the lowest cost at which the process could be operated, and the temperature and normality of the minimum were taken as the conditions for this minimum cost.

Having found the optimum conditions, one could determine for the entire process sizes and capacities of equipment, quantities of materials used, and conditions at all points in the process.

RESULTS

Converter
- Temperature: 327°F
- Acid normality: 0.010
- Length of converter: 494 ft
- Material: 10-in. stainless steel pipe
- Velocity: 0.403 ft./sec
Cooler
- Initial temperature: 329°F
- Final temperature: 160°F
- Capacity: 7,840 lb. water vaporized/hr
Filter
- Capacity: 400 sq. ft. filtering area/unit
- Number of units: 2
- Filtering rate: 5.8 gal./(hr.)(sq.ft.)
Ion Exchangers
- Anion tower
  - Height: 11.60 ft
  - Diameter: 3.92 ft
  - Resin capacity: 64.3 cu.ft. resin
  - Tower velocity: 6 gal./(min.)(sq.ft. of tower)
- Cation tower
  - Height: 13.0 ft
  - Diameter: 4.52 ft
  - Resin capacity: 96.4 cu.ft. resin
  - Tower velocity: 4.53 gal./(min.)(sq.ft. of tower)

Cost of the Process

<table>
<thead>
<tr>
<th>Component</th>
<th>Cost</th>
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<tbody>
<tr>
<td>Converter</td>
<td>$62.80</td>
</tr>
<tr>
<td>Steam</td>
<td>100.25</td>
</tr>
<tr>
<td>Acid</td>
<td>4.05</td>
</tr>
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<td>Water</td>
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<td>Cooler</td>
<td>22.65</td>
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<tr>
<td>Filter</td>
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<td>Exchange towers</td>
<td>59.40</td>
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<td>Exchange resins and</td>
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<tr>
<td>regeneration materials</td>
<td></td>
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<tr>
<td><strong>Total cost</strong></td>
<td>$604.15/day</td>
</tr>
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</table>

Cost/100 lb. of dry substance produced: $0.2012
SAMPLE CALCULATIONS

Calculations are based on optimum conditions.
Supply slurry 0.1% ash
23°Be.
% dry substance = 40.87
lb./gal. = 9.900
lb. dry substance/gal. = 4.046
Desired hydrolysis from converter 15.14°Be.
% dry substance = 26.9
lb./gal. = 9.300
lb. dry substance/gal. = 2.502
75.0% dextrose on dry basis
From Figure 4 of problem statement
F = 0.9189 for 75% dextrose.
Desired slurry entering converter
2.502(0.9189) = 2.299 lb. dry substance/gal.
% dry substance = 24.93
lb./gal. = 9.222
From Figure 1 of problem statement \( X = 81.8\% \) for hydrolysis concentration = 24.93%

\[
\therefore \quad K_1 \cdot t = 2.3(0.818) \log \left( \frac{0.818}{0.818 - 0.750} \right)
\]

\[
K_1 \cdot t = 2.04
\]

Plant Capacity
Final = 300,000 lb. dry substance/day
Before ion exchanger = \( \frac{300,000}{0.96} \)
= 312,500 lb. dry substance/day
Before filter = \( \frac{312,500}{0.97} \)
= 322,000 lb. dry substance/day
Entering converter = 322,000(0.9189) =
296,000 lb. starch/day
\[
\frac{296,000}{0.4087} = 724,000 \text{ lb. slurry required/day}
\]
\[
\frac{296,000}{0.2493} \times 0.7507 = 882,000 \text{ lb. water in hydrolyzate/day}
\]

Converter
\[
327 = 787^\circ R.
\]
\[
\frac{1}{R} = 1.270 \times 10^{-3}
\]
\[
\frac{K_1}{N} = 10.00 \quad \frac{K_2}{N} = 1.85
\]
\[
K_1 = 0.10 \quad K_2 = 0.0185
\]

\[
\therefore \quad t = 20.4 \text{ min.} = 1,224 \text{ sec.}
\]
\[
\frac{322,000}{0.269} = 1,198,000 \text{ lb. hydrolyzate leaving converter/day}
\]
\[
= 13.88 \text{ lb. hydrolyzate/sec.}
\]
13.88(0.01773) = 0.221 cu.ft./sec.  
0.01773 = cu.ft./lb. water at 327°F.  
1.1168 = sp.gr. of exit hydrolyzate

10-in. pipe  
A = 0.548 sq.ft.

Velocity = \frac{0.221}{0.548} = 0.403 ft./sec.

Length of converter = 0.403(1224) = 493.5 ft.

Length of one piece of pipe = 30 in.
Length of one bend of pipe = 2.62 in.

No. of lengths of pipe = \frac{493.5}{30 + 2.62} = 15.1

Length of bends = 15(2.62) = 39.2 ft.

Length of straight pipe = 454.2 ft.

15(1,090.00) = $16,380.00
454.2(132.00) = 60,000.00
$76,380.00 initial cost

$26,380.00(0.25) \div 300 = $62.80/day

Steam (332°F.)
Steam = 1,192.3 - 302.8 = 889.5 B.t.u./lb.
Hydrolyzate = 296,000 (0.30)(332 - 100) = 20,600,000
882,000 - X (1.00)(332-100) = 204,500,000 - 232X
\frac{225,100,000 - 232X}{225,100,000 - 232X} B.t.u./day

Steam required = 889.5X = 225,100,000 - 232X
X = 200,500 lb./day

Cost of steam = $0.50(200.5) = $100.25/day

Acid 0.010 N
0.010 N = 0.00049 lb. acid/lb. water
$18.00/2,000 lb. 96% acid = $18.00/1,920 lb. acid
= $0.00937/lb. acid
0.00049($0.00937)(882,000) = $4.05/day

Water
882,000 lb. water in hydrolyzate/day

\frac{296,000 \times 0.5913}{0.4087} = 430,000 lb. water from slurry/day

882,000 - 430,000 = 453,000 additional water required/day
200,500 lb. from steam
252,500 lb. water net required/day

252,500 \frac{0.02}{1.005 \times 8.328 \times 1,000} = $0.603/day
Cooler \((322^\circ F.)\) \quad \text{Steam }160^\circ F. = 1,130.2 \text{ B.t.u./lb.}
\begin{align*}
1,130.2 & - 292.4 = 837.8 \text{ B.t.u./lb. water vaporized} \\
0.30(0.269)(322 - 160) & = 13.08 \\
1.00(0.731)(322 - 160) & = 118.40 \\
131.48 & = 0.157 \text{ lb. water vapor/lb. hydrolyzate}
\end{align*}
\[ \frac{1,198,000(0.157)}{24} = 7,840 \text{ lb. water vapor/hr.} \]
\[ \frac{20,000(7,840)}{4,700} \times 0.6 = \frac{27,200.00 \text{ initial cost}}{300} = \frac{22.65}{\text{day}} \]

Filter
\[ \frac{882,000 - 188,000}{322,000\text{ lb. dry substance/day}} = \frac{694,000 \text{ lb. water/day}}{1,016,000 \text{ lb. hydrolyzate/day}} \]
\[ \frac{322,000}{1,016,000} = 31.7\% \text{ dry substance} \]

Hydrolyzate
\[ 17.84^\circ \text{ Be.} \]
\[ 3.011 \text{ lb. dry substance/gal.} \]
\[ \text{Filter rate } = 5.8 \text{ gal./(hr.)(sq. ft.)} \]
\[ \frac{1.023(13,420)}{3.011(5.6)} = 816 \text{ sq. ft.} \]
\[ 1.023 = \frac{60}{160} \]

To get a filtering surface of 816 sq. ft., three filters must be used. If only 800 sq. ft. or less was needed, two filters could be used. The two large filters are cheaper than three smaller ones; therefore, it is desirable to drop required surface to below 800 sq. ft. This can be accomplished by adding water before filtration. Add 4,000 lb. water/day.
\[ \frac{222,000}{1,020,000} = 31.5\% \text{ dry substance} \]

Hydrolyzate
\[ 17.78^\circ \text{ Be.} \]
\[ 2.989 \text{ lb. dry substance/gal.} \]
\[ \text{Filter rate } = 5.8 \text{ gal./(hr.)(sq. ft.)} \]
\[ \frac{1.023(13,420)}{2.989(5.8)} = 794 \]
\[ \therefore 800 \text{ sq. ft. required} \]

Two 400-sq. ft. filters
\[ \frac{41,000.00(400)}{250} \times 0.6 = \frac{54,400.00(0.25)}{300} = \frac{45.40}{\text{day @}} \]
\[ 400(0.27)(\$0.0185) = 2.00/hr \]
\[ \frac{48.00}{\text{day @}} = \frac{\text{total cost of filters}}{2(45.40 + 48.00)} = \frac{186.40}{\text{day}} \]

Ion Exchanger
Cost/regeneration/cu. ft. resin

Anion
\[ \text{Replacement } \frac{50.00}{500} = 0.1000 \]

\[ \text{Chemicals } \frac{(2.1)(93.00)}{2,000} = 0.0976 \]

Fig. 10.
Process water \[ \frac{20(\$0.02)}{1,000} = 0.0004 \]

Demineralized water \[ \frac{167.5(\$0.235)}{1,000} = 0.0394 \frac{\$0.2374}{\text{regeneration/cu.ft.}} \]

Cation Replacement \[ \frac{20.00}{500} = 0.0400 \]

Chemicals (H\(_2\)SO\(_4\)) \[ \frac{(5.0)(18.00)}{2,000(0.96)} = 0.0469 \]

Process water \[ \frac{20(\$0.02)}{1,000} = 0.0004 \]

Demineralized water \[ \frac{64.5(0.235)}{1,000} = 0.0152 \frac{\$0.1025}{\text{regeneration/cu.ft.}} \]

Time Required for Regeneration

Anion

Backwash = 16 min.
Rinse = 150
Sweetening = 6
Regeneration = 40
Depressing liquid level = 10

Total = 222 min. = 3.6 hr.

Cation

Backwash = 16 min.
Rinse = 30
Sweetening = 6
Regeneration = 40
Depressing liquid level = 10

Total = 102 min. = 1.7 hr.

Since columns are regenerated in pairs, time is determined by anion tower.
Cation column replaced every 3.6 hr.
Anion column replaced every 7.2 hr.

Absorbing Rates

Anion

\[ \frac{2.5}{\frac{7.2}{7.2}} = 0.347 \text{ lb. H}_2\text{SO}_4/\text{cu.ft./hr.} \]

Cation

\[ \frac{1.0}{\frac{3.6}{3.6}} = 0.278 \text{ lb. Na}_2\text{SO}_4/\text{cu.ft./hr.} \]

\[ \frac{0.2}{\frac{3.6}{3.6}} = 0.0556 \text{ lb. H.M.F.}/\text{cu.ft./hr.} \]

Columns of Ion Exchangers

Concentration of H.M.F.

\[ y = 0.0185 \left( \frac{20.4 - 0.75}{0.10} \right) = 0.2386 \%
\]

Impurities Needed to Be Removed

H.M.F. = (0.000385)(312, 500)
Na\(_2\)SO\(_4\) from slurry
Na\(_2\)SO\(_4\) from water add. 256, 500 (80 \(\frac{1}{(7,000)(1.005 \times 8,328)}\))

H\(_2\)SO\(_4\) total = 638 + 432

\[ = 120.5 \text{ lb./day} \]

\[ = 296.0 \text{ lb./day} \]

\[ = 342 \text{ lb./day} \]

\[ = 1,070 \text{ lb./day} \]
Cation column

\[
\frac{5.02}{0.0556} = 90.3 \text{ cu. ft. resin for } H.M.F.
\]

\[
\frac{26.8}{0.278} = 96.4 \text{ cu. ft. resin for } Na_2SO_4
\]

\[
\therefore \quad 96.4 \text{ cu. ft. used}
\]

\[96.4(0.1025)(6.67) = \$65.80 \text{/day}\]

\[\$51,000 \left( \frac{96.4}{300} \right)^{0.6} = \$25,800.00 \text{ initial cost}\]

\[\frac{\$25,800(0.25)}{300} = \$21.50 \text{/day}\]

Anion column

\[
\frac{44.6}{0.347 \times 2} = 64.3 \text{ cu. ft. resin for } H_2SO_4
\]

\[64.3(0.2374)(6.67) = \$101.80 \text{/day}\]

\[\$115,000 \left( \frac{64.3}{300} \right)^{0.6} = \$45,600.00 \text{ initial cost}\]

\[\frac{\$45,600(0.25)}{300} = \$37.90 \text{/day}\]

Column size

\[
322,000 - 312,500 = 9,500 \text{ lb. dry substance lost in filtration}
\]

\[9,500(1.79) = 17,050 \text{ lb. water lost}\]

\[698,000 - 17,050 = 680,950 \text{ lb. water after filtration}\]

\[312,500 \text{ lb. dry substance after filtration}\]

\[993,400 \text{ lb. of hydrolyzate/day}\]

\[
\frac{312,500}{993,400} = 31.50\%
\]

\[9.526 \text{ lb./gal.}\]

\[
\frac{1.005(993,400)}{9.526(24)} = 4,370 \text{ gal./hr.}
\]

\[= 72.9 \text{ gal./min.}\]

\[
\frac{72.9}{6} = 12.13 \text{ sq.ft. of tower min.}
\]

\[
\frac{64.3}{12.13} = 5.30 \text{ ft. bed depth}
\]

Column height=11.60 ft. Column diameter=3.92 ft.

Cation column

\[
\frac{72.9}{6} = 12.13 \text{ sq.ft. of tower at max. velocity}
\]

\[
\frac{96.4}{6} = 16.1 \text{ sq.ft. of tower at max. bed depth}
\]

\[
\frac{72.9}{16.1} = 4.53 \text{ gal./(min.)/(sq.ft.) velocity}
\]

Column height=13 ft. Column diameter=4.52 ft.

Total Cost

<table>
<thead>
<tr>
<th>Item</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
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<td>Converter</td>
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<td>Steam</td>
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<tr>
<td>Acid</td>
<td>4.05</td>
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<tr>
<td>Water</td>
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<td>Cooler</td>
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<tr>
<td>Columns</td>
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<tr>
<td>Resins and regenerator</td>
<td>167.60</td>
</tr>
</tbody>
</table>

\[\$604.15 \text{/day}\]

Cost of production/100 lb. dry substance=\$0.2012

Temperature = 327°F. Normality = 0.010N