MILP - A VERSATILE METHOD FOR PRODUCTION OPTIMIZATION

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Agenda

- Introduction
  - Optimization
  - Modeling philosophy
- Examples
  - Single time period modeling
    1. Simple Boiler
    2. Purchased Fuel Cost
  - Multi time period modeling
    3. Inventory
    4. Start up delay
- Conclusions
- References
- Q & A

Introduction : optimization

- Optimization: is a method for determining the "best" for a "system".
  - "Best" according to a specified criteria (or objective function)
    - Ex: Maximize Profit, Minimize Cost etc.
  - "System"
    - Described by a set of equations (or constraints)
- Two categories: Design Optimization & Operations Optimization
- Production Optimization: is Operations Optimization of a production system.
  - Production System: refers to a larger scale system
    - Ex: A plant utility system, a production plant, not a unit op

Introduction : optimization

Popular optimization methods are:

– LP	global optimum
– QP	local optimum
– NLP	local optimum
– MILP	global optimum
– MIQP	local optimum
– MINLP	local optimum
– SLP	local optimum
– SQP	local optimum

Introduction : models

- "Essentially, all models are wrong, but some are useful."
  - George E. P. Box

- We will be dealing with algebraic linear models
  - a X + b Y … ⩽ c   or   a X + b Y … - c ⩽ 0
    - a, b, c … are constants
    - X, Y, Z … are variables.
      - Most variable values are floating point (1.23)
      - Some variable values restricted to be binary (0/1)

Introduction : modeling philosophy

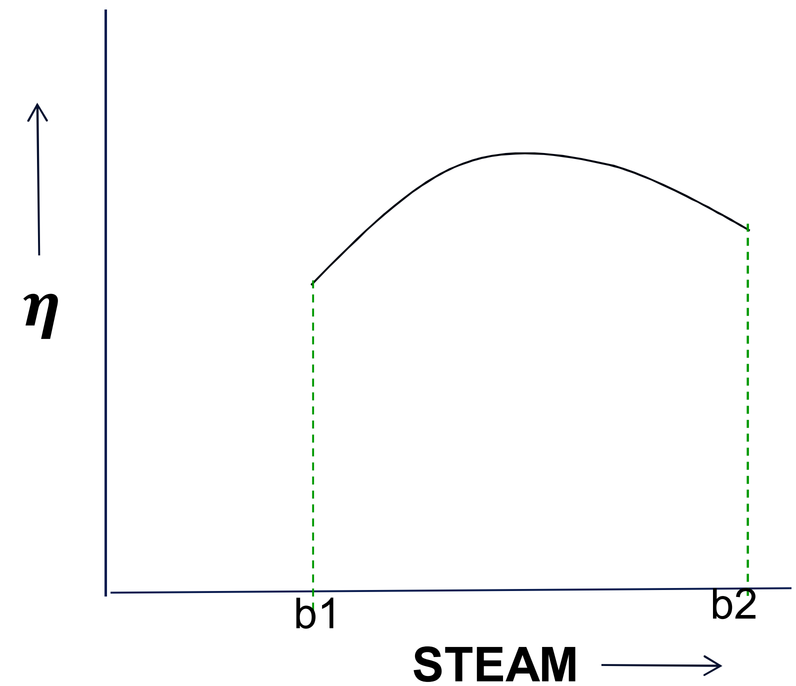
- Keep the scope as large as possible
  - Preferably the entire system
- Keep modeling as simple as possible … but not any simpler [1]
  - Start simple
  - Add complexity as needed

[1] "Everything should be made as simple as possible, but not simpler"

- Albert Einstein rephrasing Occam's Razor

Example 1: Modeling a Simple Boiler

Conventional Wisdom is to use efficiency curves



- Usually narrow efficiency range over normal operating range
- Usually approximated by a quadratic

Is there a problem?

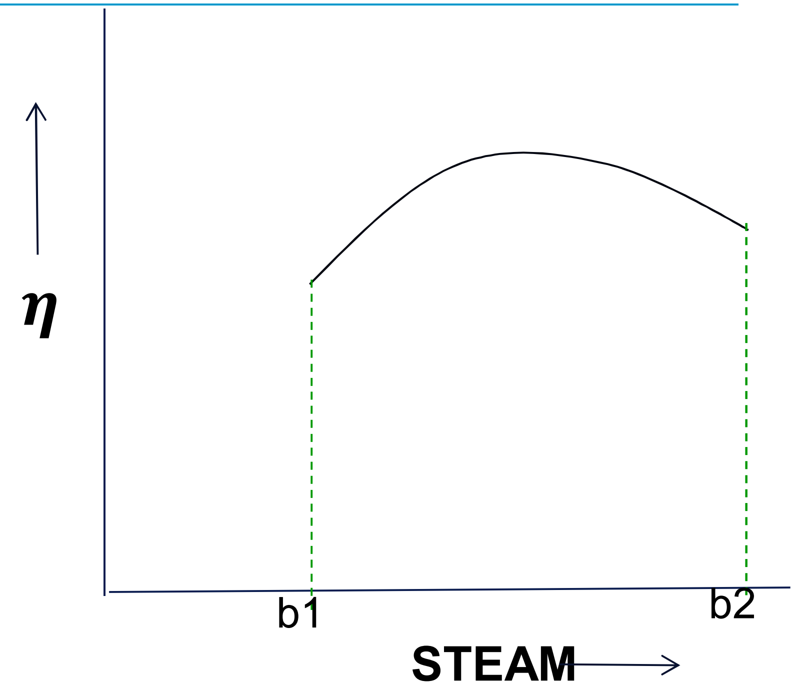
Example 1: Modeling a Simple Boiler

- Efficiency is

$$= \frac{\text{energy} \quad \text{absorbed}}{\text{energy} \quad \text{supplied}}$$

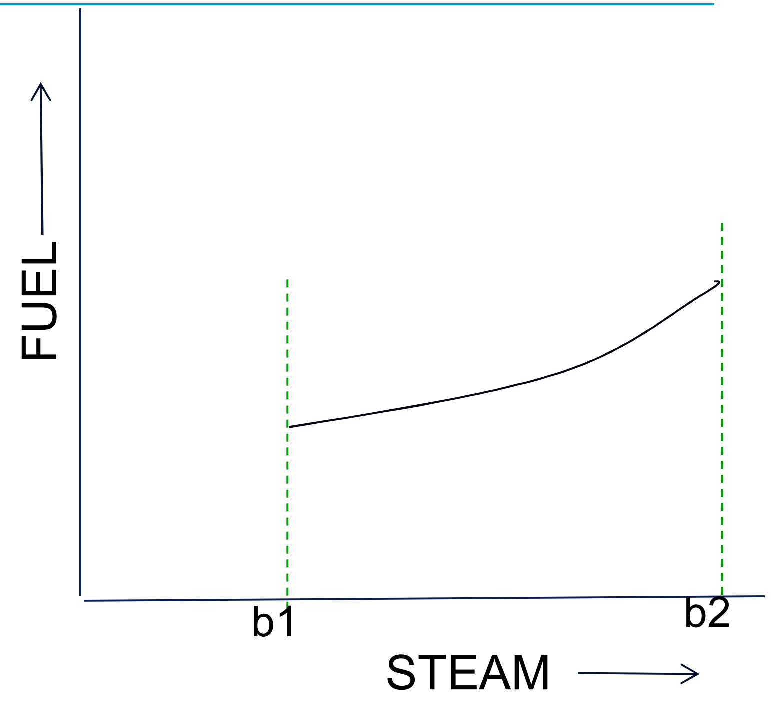
$$= \frac{\Delta\, Hv \quad *\ Steam}{\Delta\, Hc \quad *\ Fuel}$$

$$= \alpha \ * \frac{Steam}{Fuel}$$



- The relationship has only 2 variables:
  - STEAM
  - FUEL
- Plot FUEL vs. STEAM instead

Example 1: Modeling a Simple Boiler



- This relationship is
  - Considerably more linear
    - is not going through a peak
    - could be approximated by linear segments
    - segment slopes are increasing with STEAM

Example 1: Modeling a Simple Boiler

For starters let us assume a linear relationship.

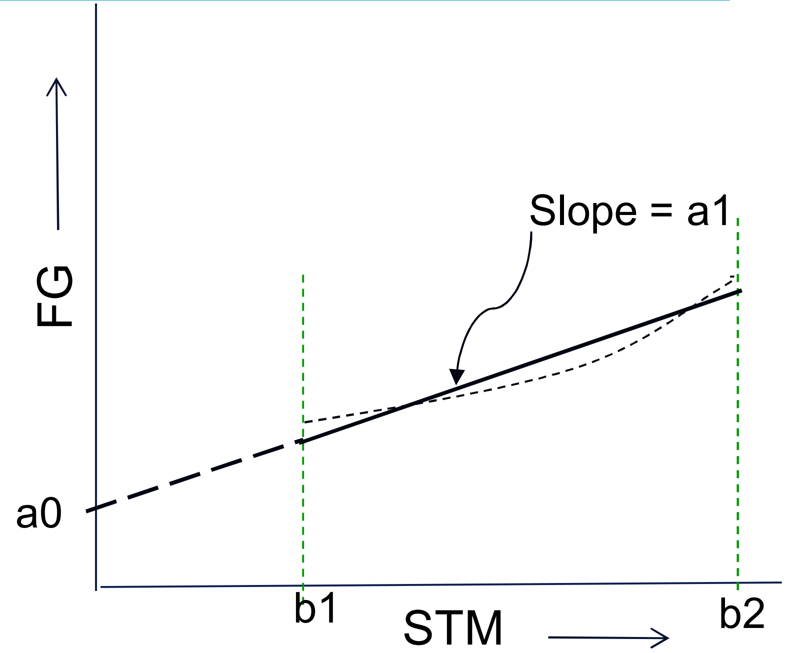
d = steam demand

Constraints:

$FG = a0 + a1 * STM$

$b1 < STM < b2$

$STM > d$



But, is there a problem?

Example 1: Modeling a Simple Boiler

- Define X, a binary variable

  X = 1 boiler is up

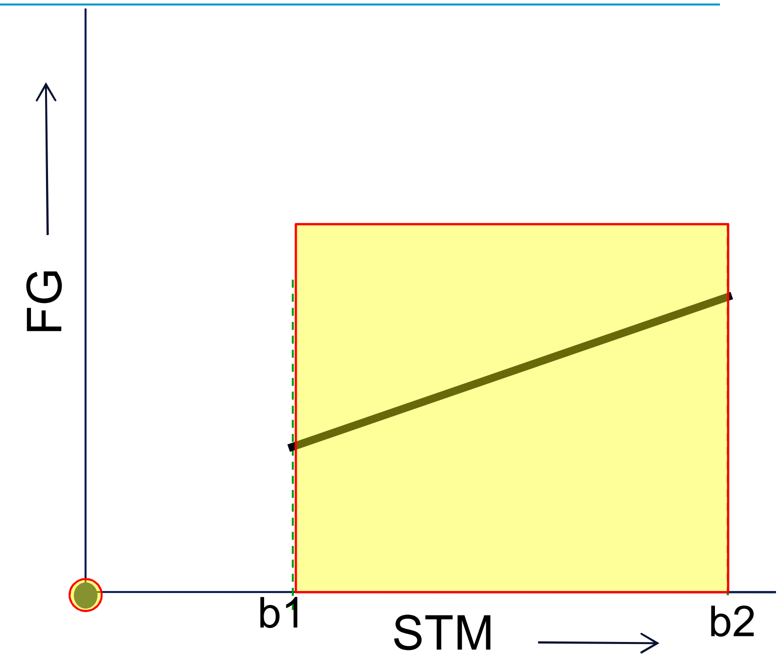
  X = 0 boiler is not up (is down)

  Revised constraints:

  FG = a0 * X + a1 * STM

  b1 * X < STM < b2 * X

  STM > d



In essence we have 2 operating regions.

But, we are not limited to 2 operating regions …

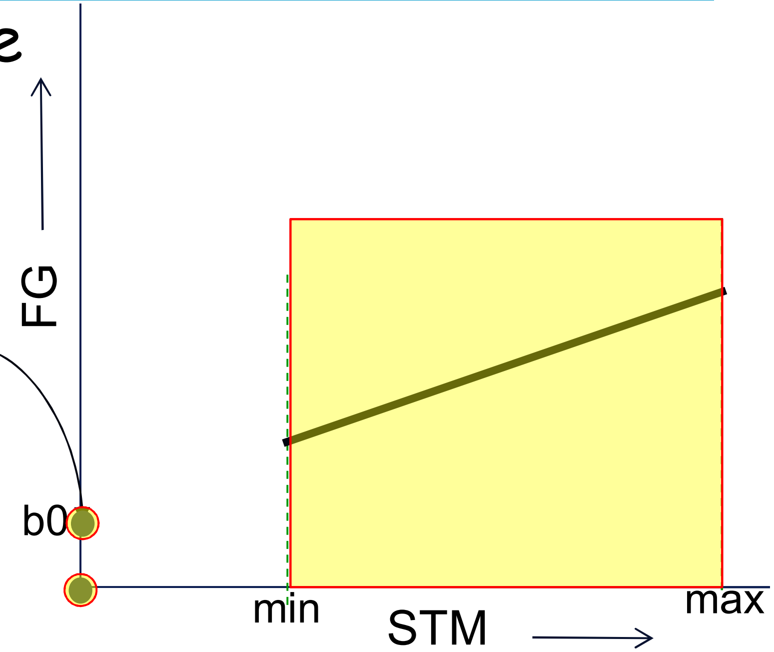
Example 1: Modeling a Simple Boiler

Define Y, another binary variable

- Y = 1 boiler in stand-by mode
- Y = 0 not in stand-by mode

Now there are 3 operating regions

		X	
		0	1
	0	Down	Up
Y	1	S/B	



Constraints:

$$FG = a0 * X + a1 * STM + b0 * Y$$

$$b1 * X < STM < b2 * X$$

$$STM > d$$

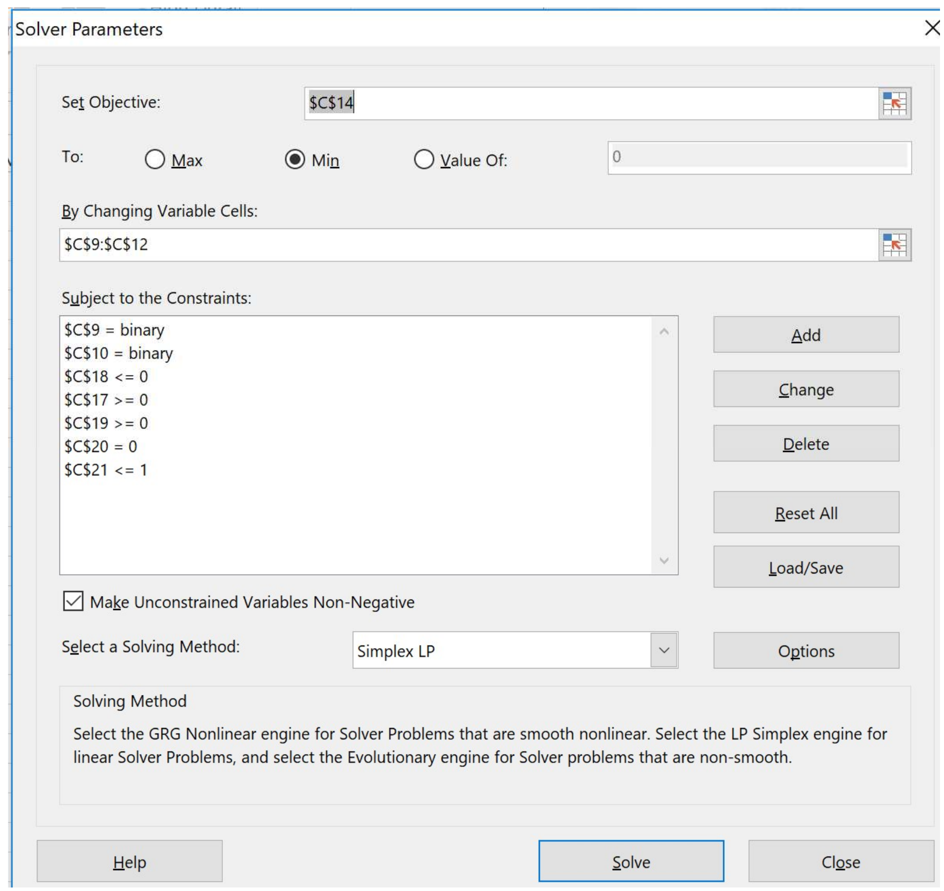
$$X + Y < 1$$

$$bin(X)$$

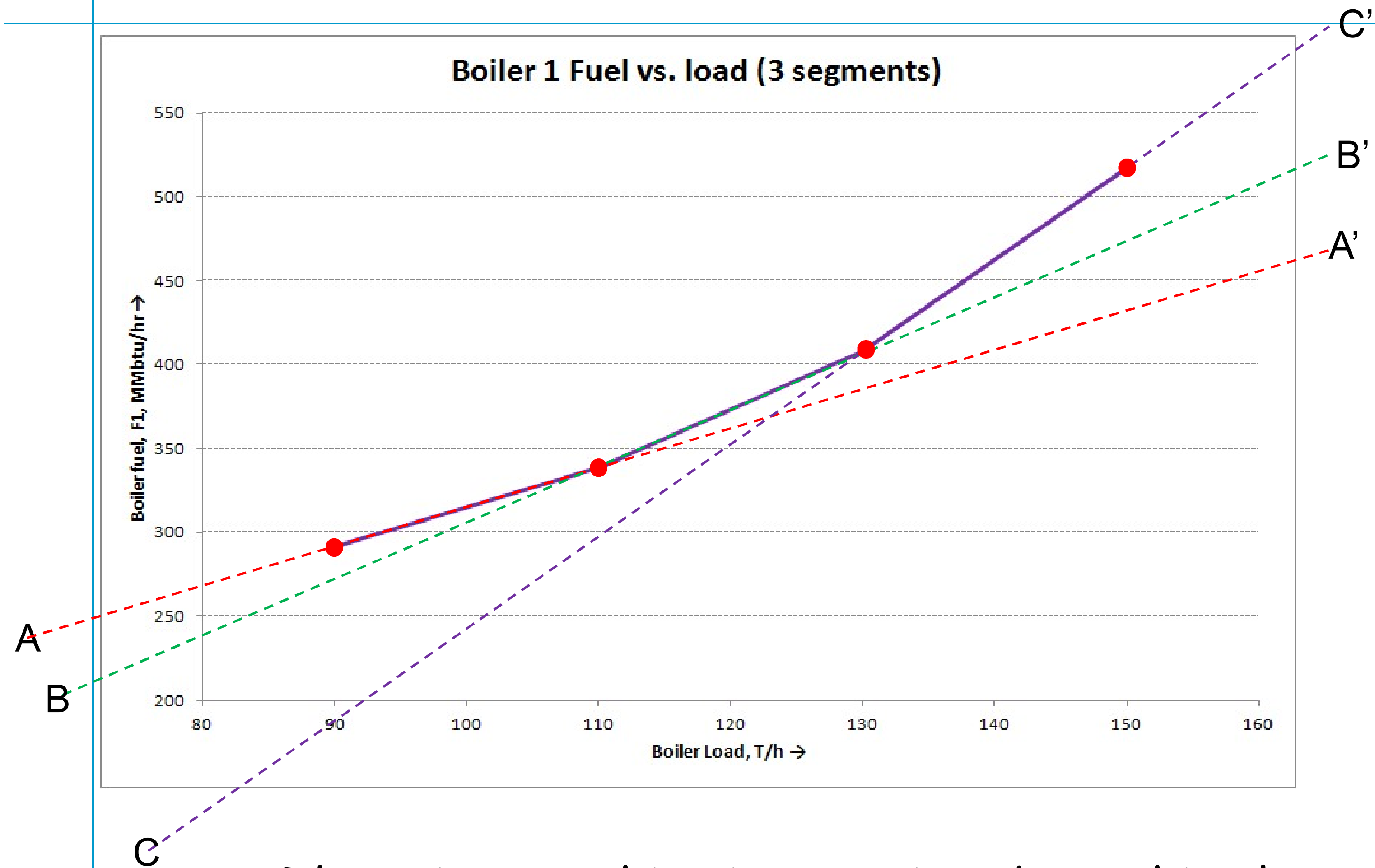
$$bin(Y)$$

Example 1: Simple Boiler : Excel Solver model

	A	B	C
1	**parameters:**	a0	1
2		a1	0.1
3		b1	50
4		b2	150
5		b0	2
6			
7	**inputs:**	d	30
8			
9	**Indep variables:**	X	**1**
10		Y	**0**
11		STM	**50**
12		FG	**6**
13			
14	**obj fn:**	FG	**6**
15			
16	**constraints:**		RHS
17	**Blr LL**	STM - b1*X ≥ 0	0
18	**Blr UL**	STM -b2*X ≤ 0	-100
19	**demand satisfaction**	STM - d ≥ 0	20
20	**FG**	FG - a0 X - b0 Y - a1 STM = 0	0
21	**Unity**	X + Y ≤ 1	1



Example 1 : Boiler example



There is something interesting about this plot:

- Each segment underestimates in the segment before and the segment after

This fact could/should be used in efficient modeling of the boiler

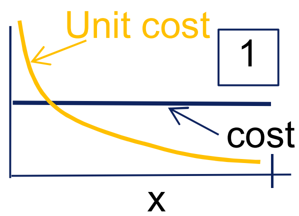
Example 2: Purchase Fuel cost

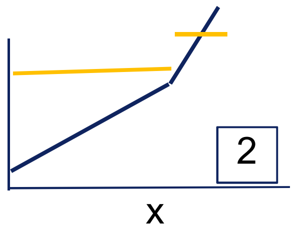
Fuel Contracts

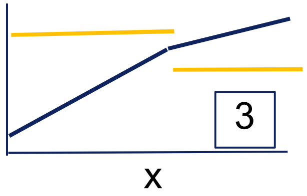
- Well defined but could be complicated

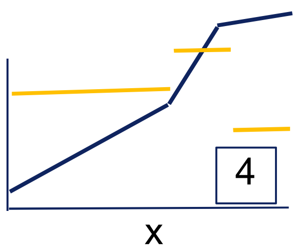
    1. Take or pay
    2. Excess penalty
    3. Volume discount
    4. Tier pricing

    ...



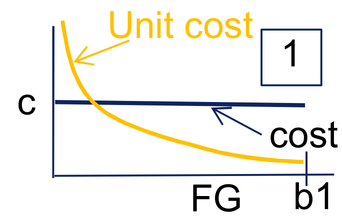






Example 2: Purchase Fuel cost (1)

1. Take or pay



b1 = contract amount of energy
c = contract fixed cost
d = fuel gas demand

FG = purchased energy
COST = energy cost

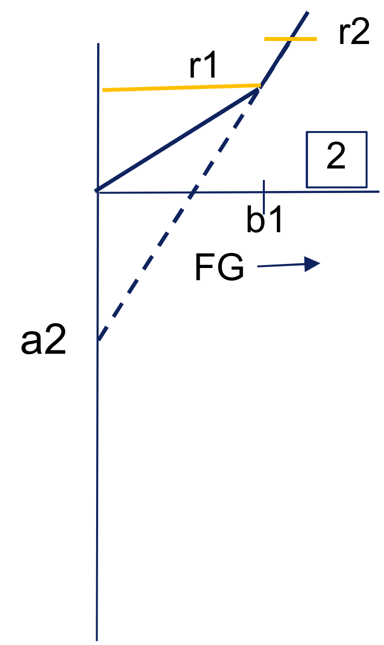
Objective: min COST
Constraints:

FG < b1
FG > d

COST = c

Example 2 : Purchase Fuel cost (2)

- Excess Penalty



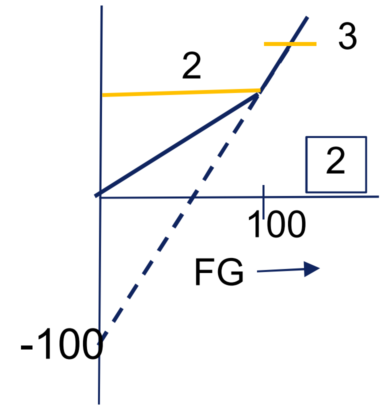
Constraints:

COST > r1 * FG

COST > r2 * FG + a2

Purchase Fuel cost (2): Excel solver

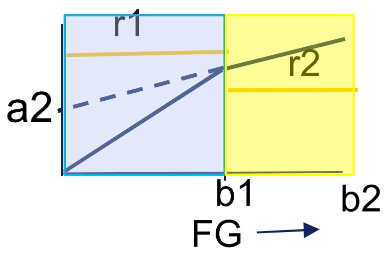
	A	B	C
1	parameters:	r1	2
2		r2	3
3		b1	100
4		a2	-100
5			
6	inputs:	demand	50
7			
8	Indep variables:	COST	**100**
9		FG	**50**
10			
11	obj fn:	COST	**100**
12			
13	constraints:		
14	**cost1**	COST - r1* FG ≥ 0	0
15	**cost2**	COST - r2* FG-a0 ≥ 0	50
16	**demand satisfaction**	FG - demand ≥ 0	0



	A	B	C
1	parameters:	r1	2
2		r2	3
3		b1	100
4		a2	-100
5			
6	inputs:	demand	150
7			
8	Indep variables:	COST	**350**
9		FG	**150**
10			
11	obj fn:	COST	**350**
12			
13	constraints:		
14	**cost1**	COST - r1* FG ≥ 0	50
15	**cost2**	COST - r2* FG-a0 ≥ 0	0
16	**demand satisfaction**	FG - demand ≥ 0	0

Example 3 : Purchase fuel cost (3)

- Volume discount



Model: define 2 regions

X1 + X2 = 1
@BIN(X1)
~~@BIN(X2)~~

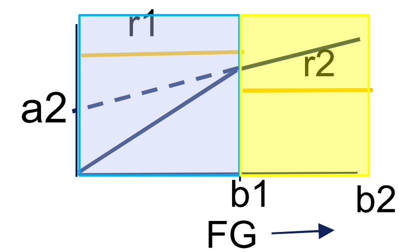
FG = FG1 + FG2

FG1 < b1 * X1
b1 * X2 < FG2 < b2 * X2

COST = r1 * FG1
+ r2 * FG2 + a2 * X2

Purchase Fuel fuel cost (3): Excel solver

	A	B	C
1	parameters	r1	2
2		r2	4
3		b1	100
4		b2	200
5		a2	-200
6			
7	inputs:	demand	50
8			
9	Indep variables:	FG1	**50**
10		FG2	**-**
11		X1	**1**
12		X2	**0**
13			
14	obj fn	COST	**100**
15			
16	constraints		
17	**Unity**	∑x -1 = 0	0
18	**FG1 UL**	b1*X1 - FG1 ≥ 0	50
19	**FG2 LL**	FG1 - b1*X2 ≥ 0	0
20	**FG2 UL**	b2*X2 - FG2 ≥ 0	0
21	**demand satisfaction**	∑FG - demand ≥ 0	0



	A	B	C
1	parameters	r1	2
2		r2	4
3		b1	100
4		b2	200
5		a2	-200
6			
7	inputs:	demand	150
8			
9	Indep variables:	FG1	**0**
10		FG2	**150**
11		X1	**0**
12		X2	**1**
13			
14	obj fn	COST	**400**
15			
16	constraints		
17	**Unity**	∑x -1 = 0	0
18	**FG1 UL**	b1*X1 - FG1 ≥ 0	0
19	**FG2 LL**	FG1 - b1*X2 ≥ 0	50
20	**FG2 UL**	b2*X2 - FG2 ≥ 0	50
21	**demand satisfaction**	∑FG - demand ≥ 0	0

Example 2 : Purchase Fuel cost (4)

Tier Pricing: define 3 operating regions

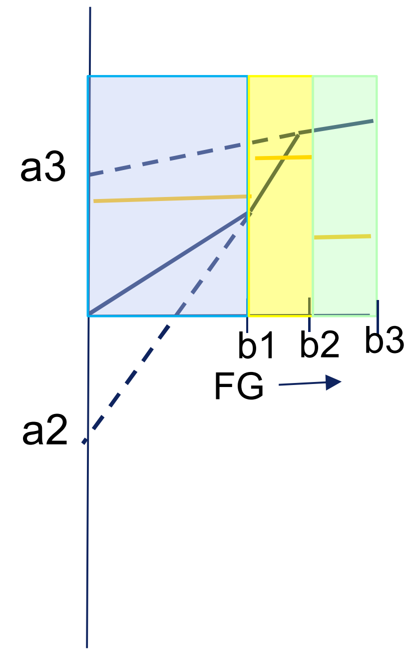
X1 + X2 + X3 = 1

@BIN(X1)
@BIN(X2)
~~@BIN(X3)~~

FG = FG1 + FG2 + FG3

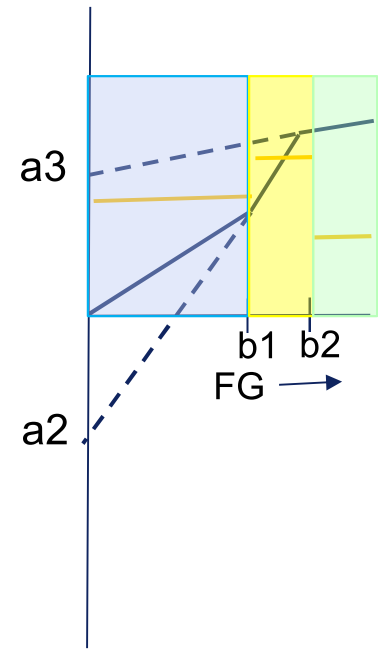
FG1 < b1 * X1
b1 * X2 < FG2 < b2 * X2
b2 * X3 < FG3 < b3 * X3

COST = r1 * FG1
+ r2 * FG2 + a2 * X2
+ r3 * FG3 + a3 * X3



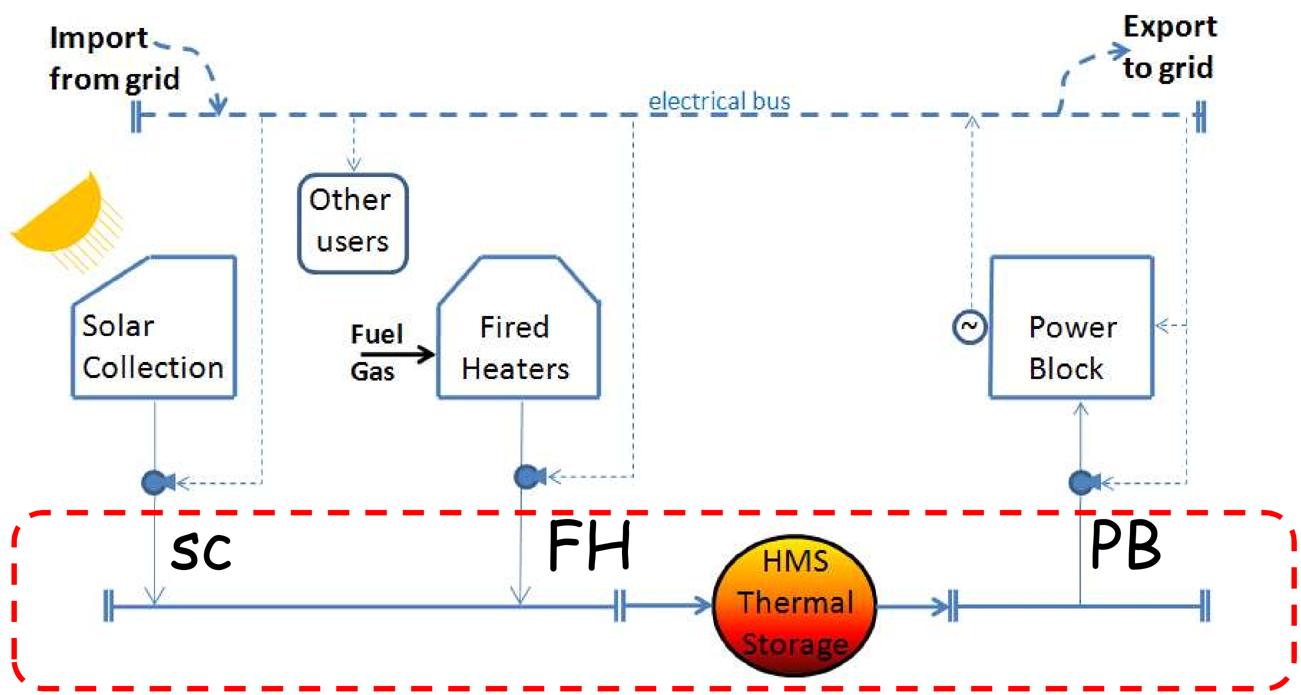
Purchase Fuel cost (4): Excel model

	A	B	C
1	parameters	r1	2
2		r2	4
3		r3	3
4		b1	100
5		b2	200
6		a2	-200
7		a3	0
8			
9	inputs:	demand	150
10			
11	Indep variables:	FG1	**0**
12		FG2	**150**
13		FG3	**0**
14		X1	**0**
15		X2	**1**
16		X3	**0**
17			
18	obj fn	COST	**400**
19			
20	constraints		
21	**Unity**	∑x -1 = 0	0.0
22	**FG1 UL**	b1*X1 - FG1 ≥ 0	0.0
23	**FG2 LL**	FG1 - b1*X2 ≥ 0	50.0
24	**FG2 UL**	b2*X2 - FG2 ≥ 0	50.0
25	**FG3 LL**	FG3 - b2*X3 ≥ 0	0.0
26	**FG3UL**	b3*X3 - FG3 ≥ 0	0.0
27	**demand satisfaction**	∑FG - demand ≥ 0	0.0



Example 3: Modeling Multi-period Inventory model

- Solar Plant



Obj fn: $\text{Max } (c_t * PB_t - 2 * FH_t)$ $t = 1 .. 24$

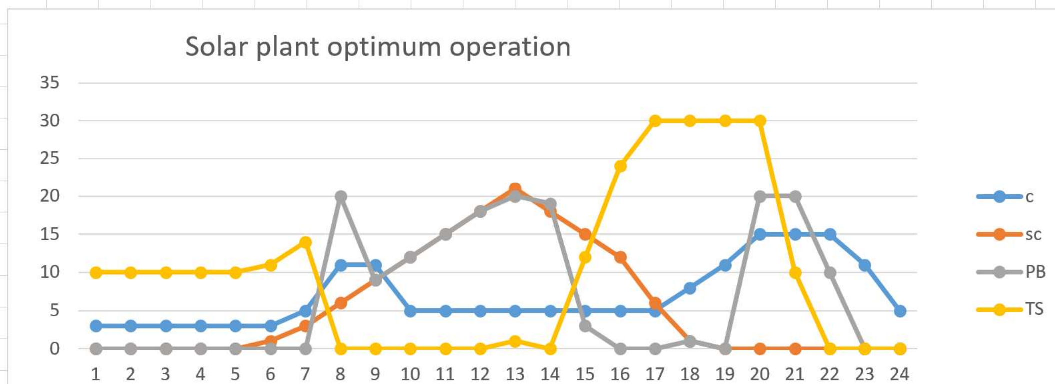
Constraints:

$TS_t = TS_{t-1} + sc_t + FH_t - PB_t$

$\sum FH < 20$ $TS_t < 30$ $PB_t < 15$

Example 3: Modeling Multi-period Inventory model

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB
1	**parameters:**	b1		20		max FH generation per day																						
2		b2		20		max PB hourly generation																						
3		b3		30		TS capacity																						
4																												
5	**inputs:**	c		3	3	3	3	3	3	5	11	11	5	5	5	5	5	5	5	5	8	11	15	15	15	11	5	
6		sc		0	0	0	0	0	1	3	6	9	12	15	18	21	18	15	12	6	1	0	0	0	0	0	0	
7																												
8			t = 0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	sum
9																												
10	**Indep variables:**	FH		**0**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	20	0	0	0	0	**20**
11		PB		**0**	0	0	0	0	0	0	20	9	12	15	18	20	19	3	0	0	1	0	20	20	10	0	0	
12																												
13																												
14	**obj fn:**	∑c * PB - 2 * FH		**0**	**0**	**0**	**0**	**0**	**0**	**0**	**220**	**99**	**60**	**75**	**90**	**100**	**95**	**15**	**0**	**0**	**8**	**0**	**260**	**300**	**150**	**0**	**0**	**1472**
15																												
16	**constraints:**																											
17	**PB capacity**	b2 - PBt > 0		20	20	20	20	20	20	20	0	11	8	5	2	0	1	17	20	20	19	20	0	0	10	20	20	
18	**TS**	TSt	10	**10**	**10**	**10**	**10**	**10**	**11**	**14**	**0**	**0**	**0**	**0**	**0**	**1**	**0**	**12**	**24**	**30**	**30**	**30**	**30**	**10**	**0**	**0**	**0**	
19	**TS capacity**	b3 - TSt > 0		**20**	**20**	**20**	**20**	**20**	**19**	**16**	**30**	**30**	**30**	**30**	**30**	**29**	**30**	**18**	**6**	**0**	**0**	**0**	**0**	**20**	**30**	**30**	**30**	
20	**sum (PB_EG)**	b1 - ∑FH > 0																										0



Example 4: Modeling boiler start up delay

- Consider a boiler with start up delay, say 1 period
  - Obviously multi period model

Base model: boiler model repeated for each time period

$X_t + Y_t < 1$

$FG_t = a0 * X_t + a1 * STM_t + b0 * Y_t$

$b1 * X_t < STM_t < b2 * X_t$

$STM_t > d_t$

How to add constraint to model delay?

Example 4: Modeling boiler start up delay

- Delay means,
  - the boiler status could be 1 in period t, if:
    - boiler was in stand-by (ie. Y = 1) in period t-1, or
    - Boiler was already up (ie. X = 1) in period t-1
  - the boiler status could <u>not</u> be 1 in period t, if:
    - boiler was completely off (ie. X = Y = 0) in period t-1

Try:

$$X_t < X_{t-1} + Y_{t-1}$$

Example 4 : Modeling boiler start up delay

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	parameters:	a0		1										
2		a1		0.1										
3		b1		50										
4		b2		150										
5		b0		2										
6														
7	inputs:	d		0	0	100	100	0	0	0	100	100	30	
8			t = 0	1	2	3	4	5	6	7	8	9	10	
9	Indep variables:	X	0	**0**	0	1	1	0	0	0	1	1	1	
10		Y	0	**0**	1	0	0	0	0	1	0	0	0	
11		STM		**0**	0	100	100	0	0	0	100	100	50	
12														sum
13	obj fn:	∑FG		**0**	**2**	**11**	**11**	**0**	**0**	**2**	**11**	**11**	**6**	**54**
14														
15	constraints:													
16	**Unity**	X + Y ≤ 1		0	1	1	1	0	0	1	1	1	1	
17	**Blr LL**	STM - b1*X ≥ 0		0	0	50	50	0	0	0	50	50	0	
18	**Blr UL**	STM - b2*X ≤ 0		0	0	-50	-50	0	0	0	-50	-50	-100	
19	**demand satisfaction**	STM - d ≥ 0		0	0	0	0	0	0	0	0	0	20	
20		Xt - Xt-1 - Yt-1 ≤ 0		0	0	0	0	-1	0	0	0	0	0	

This is a workshop

Exercises:

1. Modify the model so that the start up delay is 2 cycles (instead of 1).

   hint: Y must be 1 in the prior 2 periods before X could be 1 in the current period.

1. Further, modify the model so that each start up event incurs a fixed penalty.

2. Exercise the model and find the critical value of penalty above which the 2nd start up event becomes uneconomical.

Try these. Feel free to contact me (via e-mail) if you get stuck.

Conclusions

- MILP is a versatile modeling method especially for large systems
  - Discontinuous functions welcome (derivatives not needed)
  - Guaranteed "global" optimum
  - Fast execution
  - Mature technology
    - Robust, inexpensive, commercially available s/w
    - A limited version built into Excel
- As shown in the examples, MILP applies very well to optimization of utility systems.

References

1. Nath, R., “Real Time Optimization of an Industrial Power Plant”, AIChE Spring Meeting, San Antonio, TX (April 2013)
   - Paper discusses single time period modeling of an industrial steam plant.

2. Salbidegoitia, I. et. al., “Operations Optimization of Concentrating Solar Power Plants”, AIChE Spring Meeting, Houston, TX (April 2012)
   - Presentation discusses multi time period modeling of solar power plants.

3. Web resources (in alphabetic order)

   Commercial solver s/w companies offer fairly decent learning material. I am familiar with the following (there are probably others as well):
   - www.fico.com
   - www.lindo.com
   - www.solver.com

Q & A