

## Dragon vs Knight (author's solution)

Given:

1000 K "fire breath"

Shield: melts at 933 K,  $c_p = 903 \text{ J/Kg K}$ ,  $\rho = 2702 \text{ Kg/m}^3$ ,  $k = 237 \text{ W/m K}$ ,  
 $\alpha = 97.1 \times 10^{-6} \text{ m}^2/\text{s}$ , shield thickness  $d = 2 \text{ cm}$

Assuming: one dimensional transient conduction, constant properties, the shield can be approximated as a plate, the "knight side of the shield is adiabatic. Initially assuming lumped capacitance is not valid.

Using the method found in Incropera DeWitt Bergman and Lvine "Fundamentals of Heat and Mass Transfer" sixth edition (very similar to the method use in perry's handbook)

$$\Theta = \frac{T - T_s}{T_i - T_s} = C_1 \exp(-L_1^2 * Fo) * \cos(L_1 * x) \quad (1)$$

Where  $T$  is the temperature at some point  $x$  at some time,  $T_i$  is the initial temperature,  $T_s$  is the temperature of the surrounding fluid.

$Fo$ , the Fourier number

$$Fo = \frac{(k / (\rho * c_p)) * t}{d^2} \quad (2)$$

$C_1$  is a constant

$$C_1 = \frac{4 \sin(L_1)}{2 * L_1 + \sin(2 * L_1)} \quad (3)$$

$L_1$  is the eigenvalue, equal to the first positive root of the transcendental function

$$L_1 \tan(L_1) = Bi \quad (4)$$

Where  $Bi$  is the Biot number, which is a function of the objects characteristic length, conductivity constant and the convection constant of the surrounding fluid.

This poses something of a problem as the properties of dragon fire are not known. Assuming dragon fire had the properties of air at 1000 K a value for  $h$  was calculated using an empirical correlation of the Nusselt number. This resulted in a  $Bi = 0.002$ , this value is problematic as the table of roots I have for the eigenvalues only goes down to 0.01. In the end a Biot number of 0.01 was arbitrarily chosen.

With  $Bi$  the problem can be solved.

$$L_1 = 0.0998$$

$$C_1 = 1.00166 \sim 1$$

Theta = 0.0954

Solving equation 1 for Fo with  $x = 0$  to find the time it takes to completely melt through the shield.

Fo = 235.9

finally solving equation 2 for time (t)

t = 971.8 seconds ~ 16 minutes

Given the assumption made for Bi it is safe to assume the lumped capacitance method is valid. I would assume it would get a similar result as the one described above.